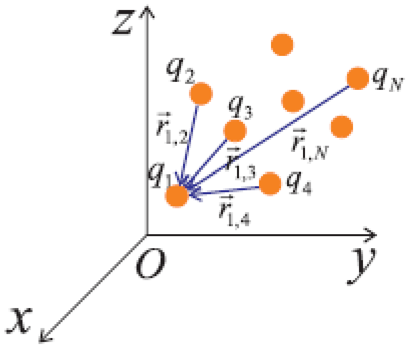
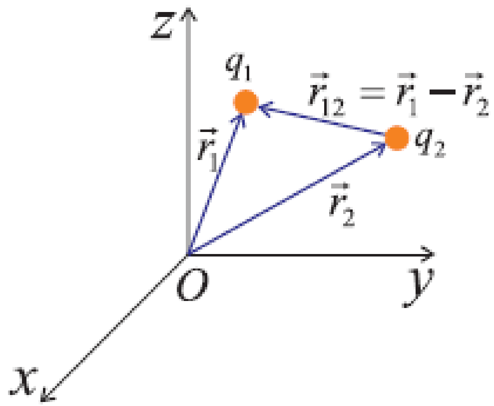
Тема 3

Електрично поле



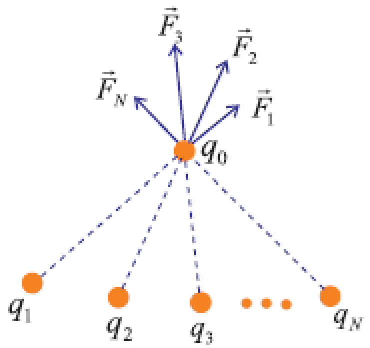
Закон на Кулон



$$\vec{F}_1 = \sum_{j=2}^{N} \vec{F}_{1,j}$$

$$\boxed{\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r_{12}^2} \cdot \hat{r}_{12}}$$

$$\hat{r}_{12} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|}$$



$$\boxed{\lim_{q_0 \to 0} \frac{\vec{F}}{q_0} = \vec{E}}$$

$= \text{Permittivity of free space} = 8.85 \times 10^{-12} C^2/Nm^2$

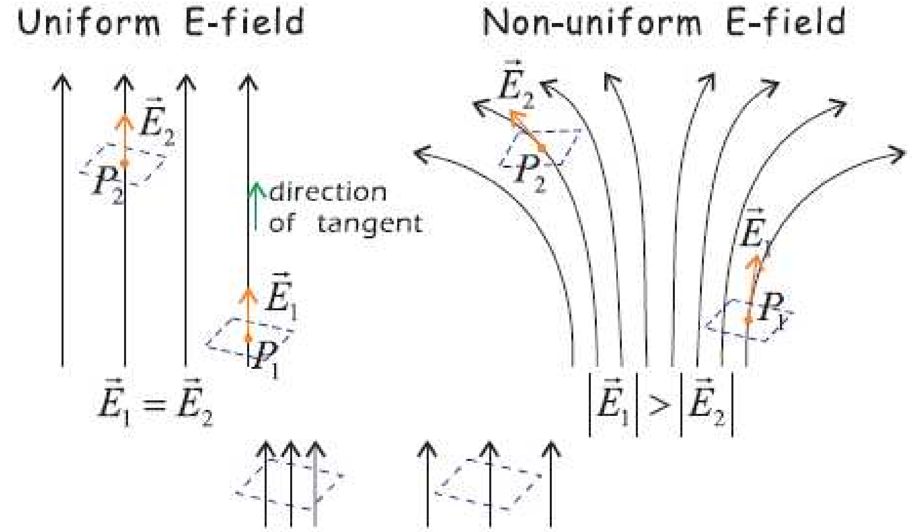
$$\boxed{\vec{F}_{21} = -\vec{F}_{12} \; \textit{Newton's 3rd Law}}$$

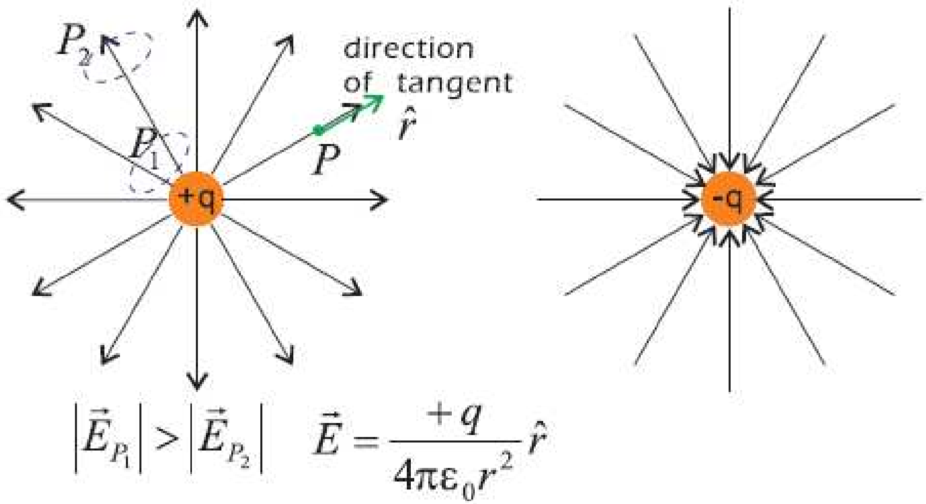
$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_N$$

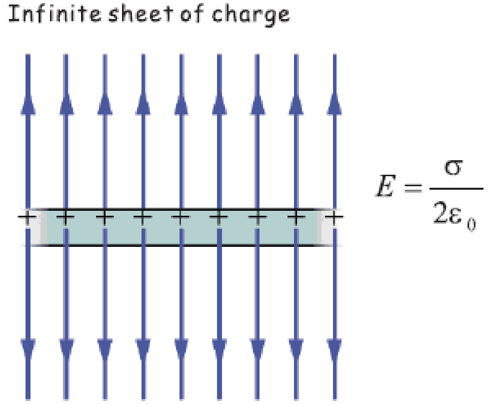
$$\boxed{\vec{E} = \sum_i \vec{E}_i = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i^2} \hat{r}_i}$$

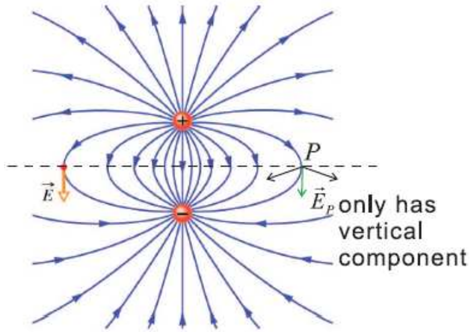
$$\vec{E}_i = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_i}{r_i^2} \cdot \hat{r}_i$$

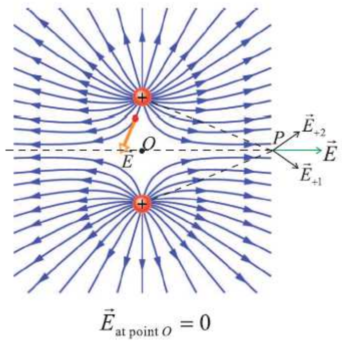
Силови линии







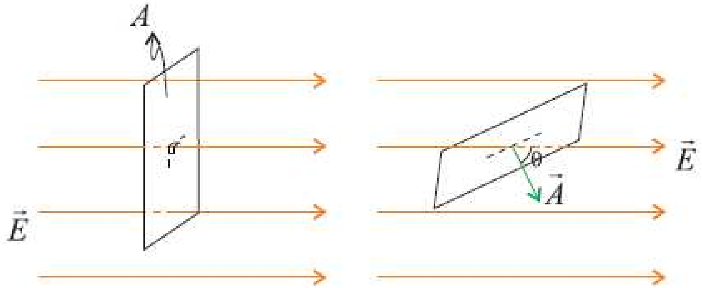




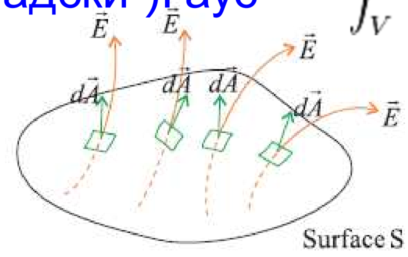
Поток на полето и теорема на (Остроградски-)Гаус

$$\int_V \operatorname{div} \vec{F}\, \mathrm{d}V = \oint_A \vec{F} \cdot \vec{n}\, \mathrm{d}A.$$

Electric flux Φ_E represents the number of E-field lines crossing a surface.



$\Phi_E = EA$ $\qquad$ $\Phi_E = \vec{E} \cdot \vec{A}$

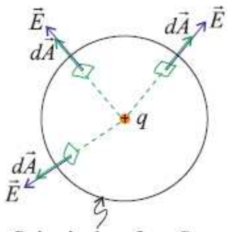


$d\vec{A}$ = Area vector for small area element dA

$$d\Phi_E = \vec{E} \cdot d\vec{A}$$

$$\Phi_E = \int_S \vec{E} \cdot d\vec{A}$$

$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A}$$



Electric flux of charge q over closed spherical surface of radius R.

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}\,\hat{r} = \frac{q}{4\pi\epsilon_0 R^2}\,\hat{r} \quad \text{at the surface}$$

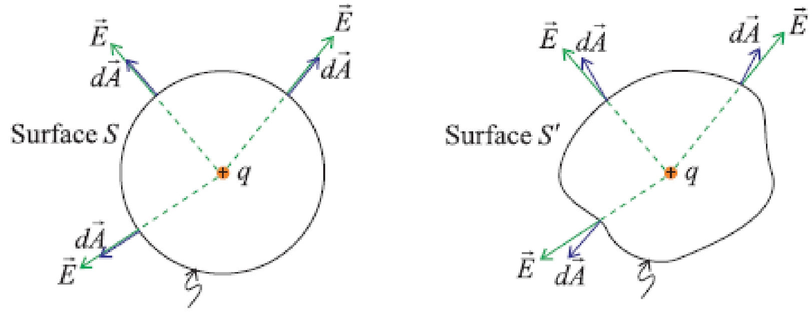
Again, $d\vec{A} = dA \cdot \hat{r}$

$$\therefore \Phi_E = \oint_S \overbrace{\frac{q}{4\pi\epsilon_0 R^2}\,\hat{r}}^{\vec{E}} \cdot \overbrace{dA\,\hat{r}}^{d\vec{A}}$$

$$= \frac{q}{4\pi\epsilon_0 R^2} \underbrace{\oint_S dA}$$

Total surface area of S $= 4\pi R^2$

$$\Phi_E = \frac{q}{\epsilon_0}$$



$\vec{E}$ // $d\vec{A}$ (// $\hat{r}$) over surface S $\qquad$ $\vec{E}$ is not // $d\vec{A}$ over surface S'

$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

Потенциал

$$U(r=\infty)=0$$

$$\begin{aligned}U_2-U_1 &= -\int_1^2 \vec{F}\cdot d\vec{r}\\ &= -\int_{r_1}^{r_2} F\,dr \quad (\because\ \vec{F}\parallel d\vec{r}\,)\\ &= -\int_{r_1}^{r_2}\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}\,dr\\ &= \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}\Big|_{r_1}^{r_2}\\ &= \frac{1}{4\pi\epsilon_0}q_1q_2\left(\frac{1}{r_2}-\frac{1}{r_1}\right)\end{aligned}$$

$$\therefore\ U_\infty - U_1 = \frac{1}{4\pi\epsilon_0}q_1q_2\left(\frac{1}{r_2}-\frac{1}{r_1}\right), \quad r_2\to\infty$$

$$\boxed{U(r)=\frac{1}{4\pi\epsilon_0}\cdot\frac{q_1q_2}{r}}$$

$$\boxed{\Delta V=\frac{\Delta U}{q_0}=\frac{-\Delta W}{q_0}}$$

$$\boxed{V=\frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i}}$$

$$\boxed{V(r)=\frac{1}{4\pi\epsilon_0}\cdot\frac{q}{r}}$$

$$\boxed{\Delta V=V_2-V_1=-\int_1^2\vec{E}\cdot d\vec{s}}$$

$$V_\infty=0 \quad\Rightarrow\quad \boxed{V_P=-\int_\infty^P\vec{E}\cdot d\vec{s}}$$

$$\boxed{E_s=-\frac{dV}{ds}}$$

$$\mathbf{E}=-\nabla V_{\mathbf{E}}.$$

$$\nabla\cdot\mathbf{E}=\nabla\cdot(-\nabla V_{\mathbf{E}})=-\nabla^2V_{\mathbf{E}}=\rho/\varepsilon_0,$$

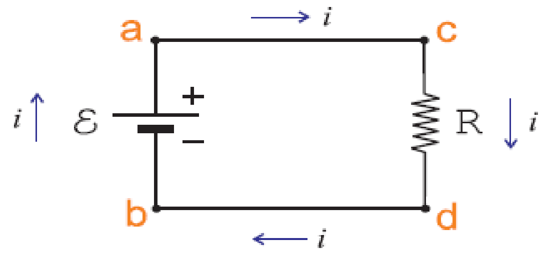
Електрически ток –

насочено движение на електрически заяди

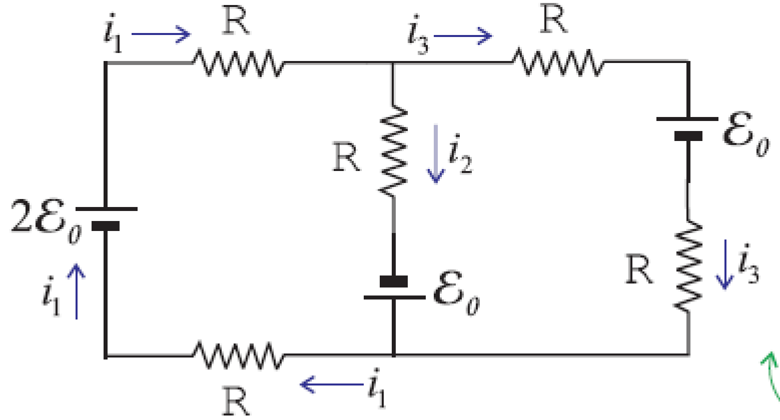
$$i = \frac{dQ}{dt}$$

$$i = \int \vec{j} \cdot d\vec{A}$$

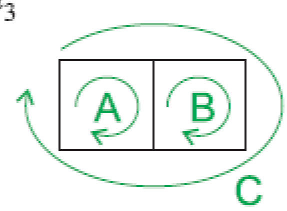
Токов контур



Закони на Кирхов:



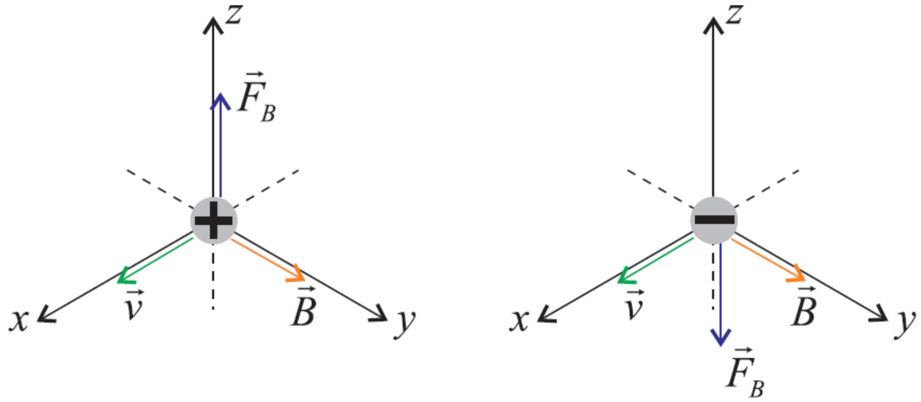
Find i_1, i_2, i_3

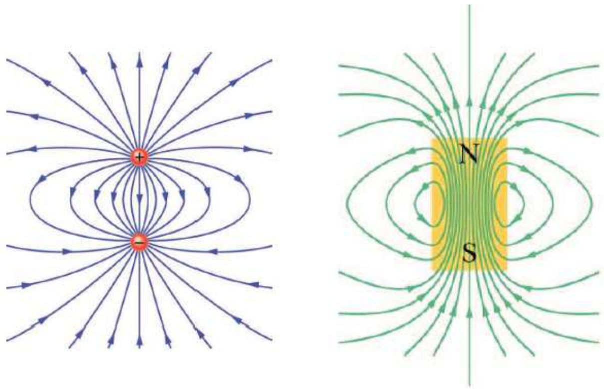


Магнитни сили и магнитно поле

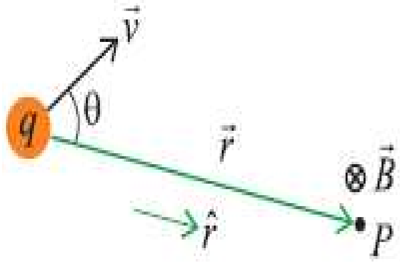
$$\vec{F}_E = q\vec{E}$$
$$\vec{F}_B = q\vec{v} \times \vec{B}$$

Сила на Лоренц:





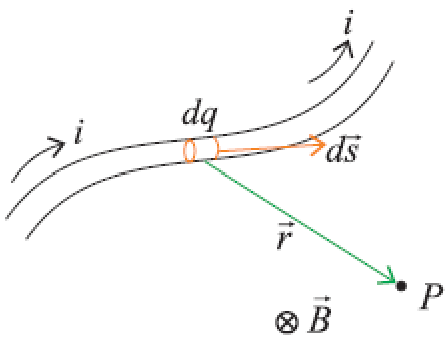
Магнитно поле на движещ се точков заряд:



$$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{q\vec{v} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \cdot \frac{q\vec{v} \times \vec{r}}{r^3}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ Tm/A } (\text{N/A}^2)$$

Магнитно поле на токов контур (Закон на Biot-Savart-Laplace:



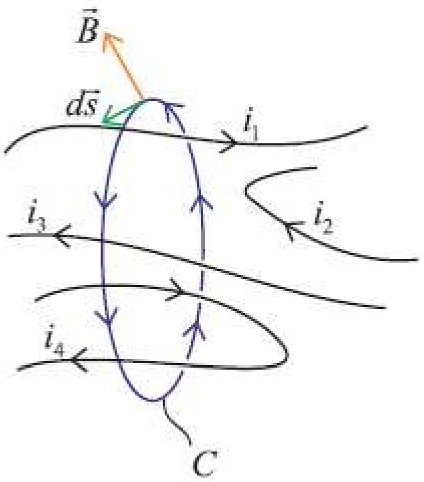
$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{i\, d\vec{s} \times \hat{r}}{r^2}$$

Теорема на Гаус:

$$\oint_S \vec{B} \cdot d\vec{A} = 0$$

Закон на Ампер:

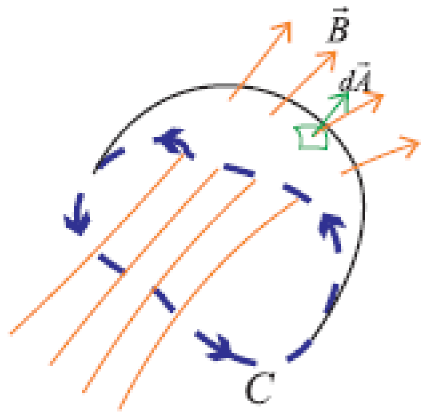
$$\oint_C \vec{B} \cdot d\vec{s} = \mu_0 i$$



$$\begin{aligned}\oint_C \vec{B} \cdot d\vec{s} &= \mu_0(i_1 - i_3 + i_4 - i_4) \\ &= \mu_0(i_1 - i_3)\end{aligned}$$

Закон на Фарадей за магнитната индукция:

$$\Phi_m = \int_S \vec{B} \cdot d\vec{A}$$



Unit of Φ_m : Weber (Wb)

$1\text{Wb} = 1\text{Tm}^2$

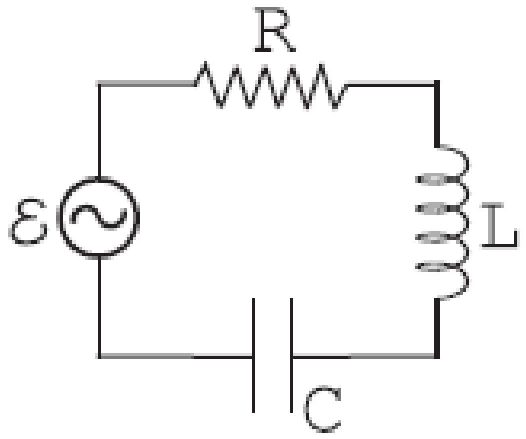
$$\text{Induced emf } |\mathcal{E}| = N \left| \frac{d\Phi_m}{dt} \right|$$

$$\mathcal{E} = -N \frac{d\Phi_m}{dt}$$

Дори и без токов контур:

$$\oint_C \vec{E}_{ind} \cdot d\vec{s} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{A}$$

Трептящ кръг



$$\mathcal{E} = \mathcal{E}_m \sin(\omega t + \delta)$$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = \mathcal{E}_m \sin(\omega t + \delta)$$

$$\boxed{i = i_m \sin(\omega t - \phi)}$$

Резонансна честота: $\boxed{\omega^2 = \frac{1}{LC}}$

$$i_m = \frac{\mathcal{E}_m}{Z}$$

Импеданс: $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

Ток на отместване:

$$\frac{dQ}{dt} = \boxed{\varepsilon_0 \frac{d\Phi_E}{dt} = i_{disp}}$$

Закон на Ампер-Максуел:

$$\boxed{\oint_C \vec{B} \cdot d\vec{s} = \mu_0 (i_{incl} + \varepsilon_0 \frac{d\Phi_E}{dt})}$$

Уравнения на Максуел:

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{Q_{incl}}{\varepsilon_0}$$

$$\oint_S \vec{B}\cdot d\vec{a} = 0$$

$$\oint_C \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{a}$$

$$\oint_C \vec{B}\cdot d\vec{s} = \mu_0 i_{incl} + \mu_0\varepsilon_0\frac{d}{dt}\int_S \vec{E}\cdot d\vec{a}$$

$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$$

Formulation in terms of *total* charge and current[note 2]

Name	Differential form	Integral form
Gauss's law	$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$	$\oiint_{\partial V} \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \frac{Q(V)}{\varepsilon_0}$
Gauss's law for magnetism	$\nabla \cdot \mathbf{B} = 0$	$\oiint_{\partial V} \mathbf{B} \cdot \mathrm{d}\mathbf{A} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint_{\partial S} \mathbf{E} \cdot \mathrm{d}\mathbf{l} = -\iint_S \frac{\partial \mathbf{B}}{\partial t} \cdot \mathrm{d}\mathbf{A}$
Ampère's circuital law (with Maxwell's correction)	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$	$\oint_{\partial S} \mathbf{B} \cdot \mathrm{d}\mathbf{l} = \mu_0 I_S + \mu_0 \varepsilon_0 \iint_S \frac{\partial \mathbf{E}}{\partial t} \cdot \mathrm{d}\mathbf{A}$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}.$$

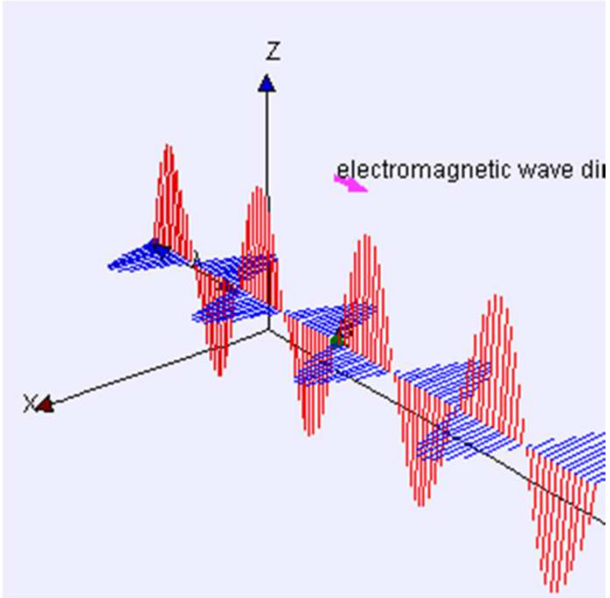
$$\mathbf{E}(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0} \int \left(\frac{\rho(\mathbf{r_s}, t_r)}{|\mathbf{r}-\mathbf{r}_s|^3}(\mathbf{r}-\mathbf{r}_s) + \frac{\frac{\partial \rho(\mathbf{r_s},t_r)}{\partial t}}{|\mathbf{r}-\mathbf{r}_s|^2 c}(\mathbf{r}-\mathbf{r}_s) - \frac{\frac{\partial \mathbf{J}(\mathbf{r_s},t_r)}{\partial t}}{|\mathbf{r}-\mathbf{r}_s| c^2} \right) \mathrm{d}^3\mathbf{r_s}$$

$$\mathbf{B}(\mathbf{r},t) = \frac{\mu_0}{4\pi} \int \left(\frac{\mathbf{J}(\mathbf{r_s}, t_r)}{|\mathbf{r}-\mathbf{r}_s|^3} \times (\mathbf{r}-\mathbf{r}_s) + \frac{\frac{\partial \mathbf{J}(\mathbf{r_s},t_r)}{\partial t}}{|\mathbf{r}-\mathbf{r}_s|^2 c} \times (\mathbf{r}-\mathbf{r}_s) \right) \mathrm{d}^3\mathbf{r_s}$$

where $t_r = t - \frac{|\mathbf{r}-\mathbf{r}_s|}{c}$ (the retarded time).

Вълнови уравнения:

$$\nabla \cdot \mathbf{E} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$



$$\nabla \times \nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\nabla \times \mathbf{B} = -\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$$
$$\nabla \times \nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\nabla \times \mathbf{E} = -\mu_o \varepsilon_o \frac{\partial^2 \mathbf{B}}{\partial t^2}$$
$$\nabla \times (\nabla \times \mathbf{V}) = \nabla (\nabla \cdot \mathbf{V}) - \nabla^2 \mathbf{V}$$

$$\mathbf{E}(\mathbf{r},t) = g(\phi(\mathbf{r},t)) = g(\omega t - \mathbf{k}\cdot\mathbf{r})$$
$$\mathbf{B}(\mathbf{r},t) = g(\phi(\mathbf{r},t)) = g(\omega t - \mathbf{k}\cdot\mathbf{r})$$

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} - {c_0}^2 \cdot \nabla^2 \mathbf{E} = 0$$
$$\frac{\partial^2 \mathbf{B}}{\partial t^2} - {c_0}^2 \cdot \nabla^2 \mathbf{B} = 0$$

$$k = |\mathbf{k}| = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$
$$\mathbf{E}(\mathbf{r}) = E_0 e^{-i\mathbf{k}\cdot\mathbf{r}}$$
$$\mathbf{B}(\mathbf{r}) = B_0 e^{-i\mathbf{k}\cdot\mathbf{r}}$$

$$c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.99792458 \times 10^8 \text{ m/s}$$