Beyond the Standard Model

Lecture 3

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SOME QUESTIONS BEYOND THE SM

i) Values of couplings, masses and mixings

- Can they be computed in some new underlying theory?

ii) The origin of electroweak symmetry breaking

- Comes from an elementary Higgs? Composite? or?

iii) The fine-tuning problems:

- The cosmological constant puzzle
- The strong-CP problem
- The gauge hierarchy problem

iii) Unification with quantum gravity

- String theory?

Solutions?

- GUT
  - SU(5), SO(10) – solve number of problems
  - In contradiction with experimental data
- Froggatt– Nielsen scenario
  - Additional flavor U(1) symmetry
  - An attempt to solve the problem with fermion masses and mixings
- Fine tuning puzzle
  - More – later
- Strong CP-problem
  - Axion solution
- Technicolor
  - Quark condensate

SUSY

but before …

The root of the problems

- Let us start with Quantum Field Theory
  - definition of observable
  - description of the interactions
  - description of the fundamental particles
  - way the constants are not constants?
- The masses
  - what is a mass
  - how to solve the problem

Observables

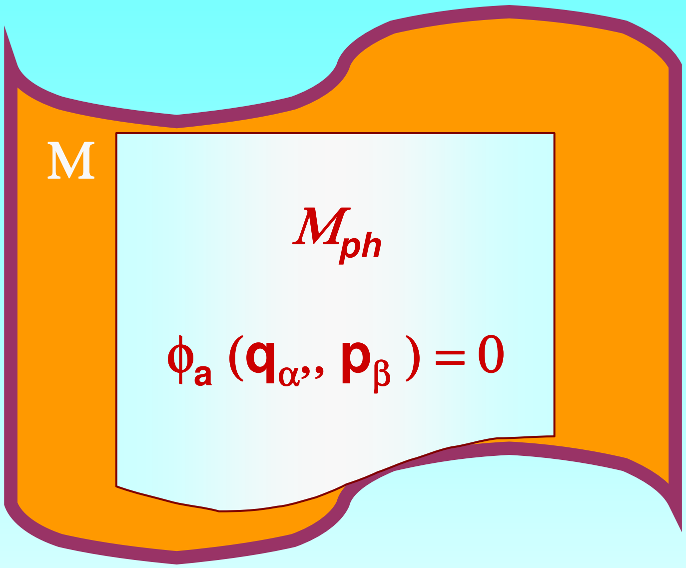
- Two definitions
  - ✓ system with coordinates and moments q_α and p_β

    phase space $\{q_{\alpha,}, p_\beta\} = M$

    Constraints $\phi_a(q_{\alpha,}, p_\beta) = 0 \;\Rightarrow\; M_{ph}$
  - ✓ *Dirac* – $f(q_{\alpha,}, p_\beta)$ is observable if

    $\{f(q_{\alpha,}, p_\beta), \phi_a(q_{\alpha,}, p_\beta)\} = 0$ holds on M_{ph}
  - ✓ *In the gauge theories*

    $f(q_{\alpha,}, p_\beta)$ *is observable if gauge invariant – holds on M*



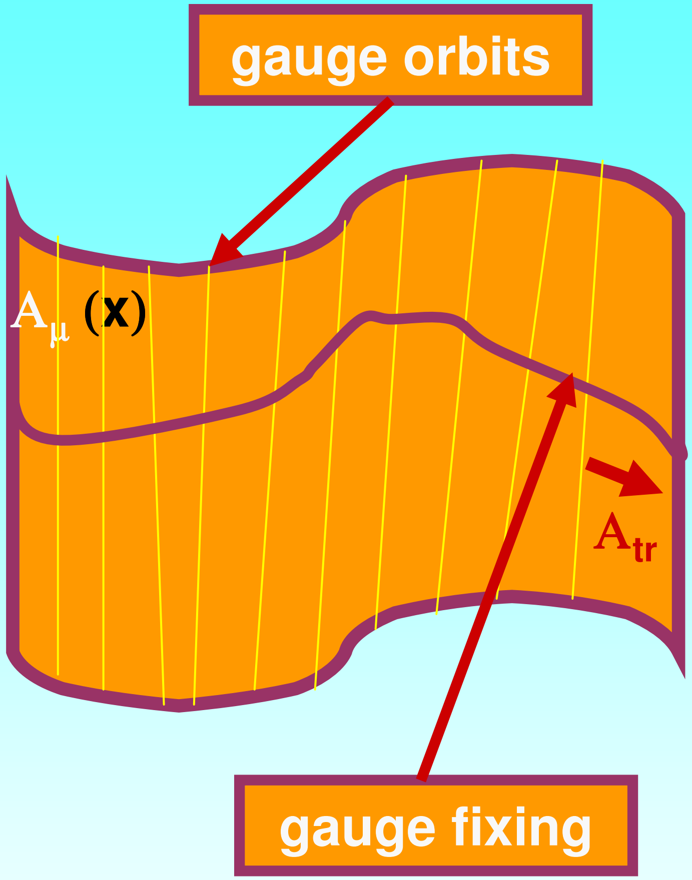
- *Equivalence*
  - ✓ *From gauge invariance follows Dirac definition*
  - ✓ *From Dirac definition* ➔ *gauge invariant only on* M_{ph}
- *In QFT – gauge invariance – probably too strong?!*

Gauge fields

- In the SM – gauge fields – spin 1,
  $m_A = 0$ ➔ two degrees of freedom
  Vector field – 4 degrees of freedom

- 1 degree – equation of motion
  for A_0 there is no dynamics –
  exclude from equation of motion

- Gauge fixing
- group orbits $\{g\}A_\mu(x)$
  for abelian group – no problem
  for non-abelian group – Gribov's ambiguity
  θ-vacuum – strong CP – violation
- A is not gauge invariant not observable
  for example A_{tr} $d_i A^i = 0$



Fundamental particles

- Spin ½ - spinor fields $\psi(x)$
- Real observable particles carry corresponding charges (electric, colour etc)
- They exit with corresponding fields surrounding them
- In QFT – naked fermions – with charges but without fields

  $\Psi'(x) = \exp(-i\varepsilon_a . J^a) . \psi(x)$ - non observable
- Constants – α, m etc corresponds to naked particles
- The field around the fermions should be taken into account somehow
- The renormalization - the effective way to come to observable physical quantities

  $\alpha(Q^2)$ – takes into account the vacuum polarization (seen by the test particle)
- $m(Q^2)$ – takes into account that the full energy is the energy of naked particle + the gauge field energy

The constants are not more constants!

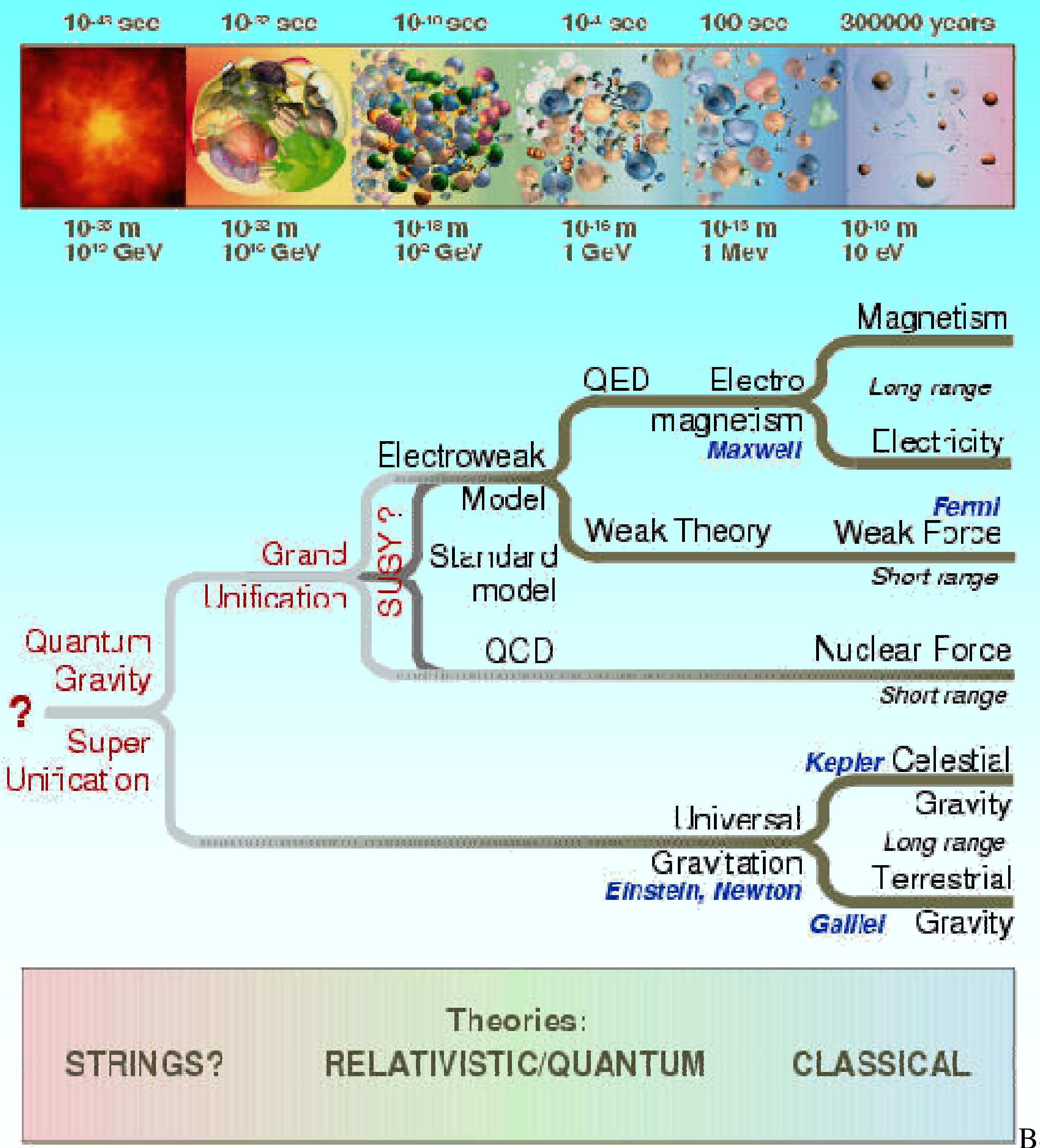
Masses

- What is a mass?
  - This physical variable is well defined only in 4D space-time
  - The mass is the aigenvalue of the Casimir operator $P^2 = m^2$

  of the Poinkare group.
- Generaly speaking – the mass and energy (and energy conservation) are something very specific for 4 – dimensional homogeneous and isotropic space-time (Minkowski space-time)
- If the space – time is with D > 4 , than the mass end the energy are not well defined variables
  - The space-time symmetry group is different ➔ the Casimir operators and conservation lows are different
- All problems in the SM are connected in one or another way with particle masses (Higgs mechanism, Yukawa couplings)

Гравитация – свойства на пространство - времето

Interactions unification



SUPERSYMMETRY

SUSY

We are looking for non trivial unification of internal and space-time symmetries

If P is the Space-time group of symmetry

$$[P_a, P_b] = 0 \qquad [P_a, J_{bc}] = (\eta_{ab} P_c - \eta_{ac} P_b)$$

$$[J_{ab}, J_{cd}] = -(\eta_{ac} J_{bd} + \eta_{bd} J_{ac} - \eta_{ad} J_{bc} - \eta_{bc} J_{ad})$$

And G is the internal symmetry group

$$[T_r, T_s] = f_{rst} T_t$$

Coulmen - Mandela No-go theorem

If P and G are Li groups, it is not possible to find group GP, for which has G $\subset$ GP and P $\subset$ GP different from G $\otimes$ P

i.e. trivial unification

The way out – use something which is not a Li group ?!

Haag – Iopushanski - Sohnius

The fine tuning problem

Two facts:

- **The mass of the minimal Standard Model Higgs boson is not far from the weak scale, ~ 200 GeV.**
- **Much larger energy scales become relevant at some point**

A first question arises: why is the weak scale so much smaller than the Planck scale, $M_P \sim 10^{19}$ GeV, or the unification scale, $M_{GUT} \sim 10^{16}$ GeV?

This is usually referred to as the hierarchy problem.

The fine tuning problem

Even worse: The Higgs mass (masses of scalar particles, in general) is strongly sensitive to any large energy scale unless a *fine tuning* of parameters is performed.

This is the so-called **naturalness problem.**

There is a very simple reason for this: scalar masses are not **naturally small**, in the sense that no symmetry is recovered when they are let go to zero.

Fermion and vector boson masses *are* naturally small: radiative corrections are proportional to the masses themselves.

The fine tuning problem

Naturalness and fine tuning in a simple example

Consider a theory of two real scalars fields:

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\,\partial_\mu\phi + \frac{1}{2}\partial^\mu\Phi\,\partial_\mu\Phi - V(\phi,\Phi)$$

with

$$V(\phi,\Phi) = \frac{m^2}{2}\phi^2 + \frac{M^2}{2}\Phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{\sigma}{4!}\Phi^4 + \frac{\delta}{4}\phi^2\Phi^2$$

Assume λ, σ, δ are all positive, small and comparable in magnitude, and assume $M^2 \gg m^2 > 0$.

Is the mass hierarchy $m^2 \ll M^2$ conserved at the quantum level?

The fine tuning problem

Compute one-loop radiative corrections to m^2 by taking the second derivatives of the effective potential at the minimum $\phi = \Phi = 0$:

$$m^2_{\text{one loop}} = m^2(\mu^2) + \frac{\lambda m^2}{32\pi^2}\left(\log\frac{m^2}{\mu^2} - 1\right) + \frac{\delta M^2}{32\pi^2}\left(\log\frac{M^2}{\mu^2} - 1\right)$$

$$\mu^2 \frac{\partial m^2}{\partial \mu^2} = \frac{1}{32\pi^2}\left(\lambda m^2 + \delta M^2\right)$$

Corrections proportional to M^2 appear at one loop. One can choose $\mu^2 \sim M^2$ in order to get rid of them, but they reappear through the running of $m^2(\mu^2)$.

The mass hierarchy is preserved only if the parameters are such that

$$\lambda m^2 \sim \delta M^2 \rightarrow \frac{\delta}{\lambda} \sim \frac{m^2}{M^2}$$

This is what we usually call a *fine tuning* of the parameters.

The fine tuning problem

The same thing happens if $m^2 < 0$, $M^2 \gg |m^2| > 0$. In this case the tree-level potential has a minimum at

$$\Phi = 0, \quad \phi^2 = -6m^2/\lambda \equiv v^2$$

and the symmetry $\phi \to -\phi$ is spontaneously broken. The degrees of freedom in this case are Φ and $\phi' \equiv \phi - v$, with

$$m_\Phi^2 = M^2 \qquad m_{\phi'}^2 = -2m^2 = \lambda v^2/3$$

At one loop, the minimization condition $m^2 + \lambda v^2/6 = 0$ is replaced by

$$m^2 + \frac{\lambda v^2}{6} = -\frac{\lambda}{32\pi^2}\left(m^2 + \frac{\lambda v^2}{2}\right)\left(\log\frac{m^2 + \frac{\lambda v^2}{2}}{\mu^2} - 1\right) - \frac{\delta}{32\pi^2}\left(M^2 + \frac{\delta v^2}{2}\right)\left(\log\frac{M^2 + \frac{\delta v^2}{2}}{\mu^2} - 1\right)$$

Following the same procedure as in the unbroken case one finds

$$m_{\phi'}^2 = \frac{\lambda v^2}{3} + \frac{v^2}{32\pi^2}\left[\lambda^2 \log\frac{m^2 + \frac{\lambda}{2}v^2}{\mu^2} + \delta^2 \log\frac{M^2 + \frac{\delta}{2}v^2}{\mu^2}\right]$$

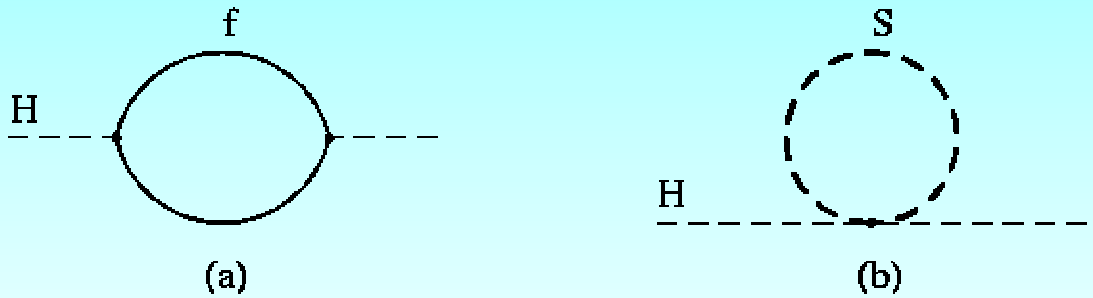
with $v \sim M$ without a suitable tuning of the parameters.

Naturalness: a closer look

The scalar potential in the Standard Model:

$$V(\phi) = m^2 \left|\phi\right|^2 + \lambda \left|\phi\right|^4$$

One-loop corrections to m^2 due to fermionic (a) or bosonic (b) degrees of freedom:



$$\left(\Delta m^2\right)_a = \frac{|\lambda_f|^2}{16\pi^2}\left[-2\Lambda^2 + 6m_f^2\,\log\frac{\Lambda}{m_f} + \ldots\right]$$

$$\left(\Delta m^2\right)_b = \frac{\lambda_S}{16\pi^2}\left[\Lambda^2 - 2m_S^2\,\log\frac{\Lambda}{m_S} + \ldots\right]$$

The fine tuning problem

Here Λ is an ultraviolet cut-off, to be identified with the energy scale at which the SM is no longer reliable, and the dots stand for terms that do not grow with Λ.

In dimensional regularization the Λ^2 term would be absent, but contributions proportional to m_f^2, m_S^2 would still be there.

Even if the heavy degrees of freedom are not directly coupled to the SM Higgs, it can be shown that similar contributions arise at higher orders.

In the absence of very special cancellations, the Higgs boson becomes as heavy as the heaviest degrees of freedom.

The fine tuning problem

A symmetry that relates fermions to bosons would do the job, at least at one loop. Suppose there are two scalars for each fermion:

$$\left(\Delta m^2\right)_{a+b} = \frac{\lambda_S - |\lambda_f|^2}{8\pi^2}\,\Lambda^2 + \ldots$$

For suitable values of the couplings the quadratic divergence disappears.

No surprise: with bosons and fermions in the same multiplet, scalar masses are protected by the same (chiral) symmetry that protects fermion masses from large radiative corrections.

Clearly, more restrictions will be needed in order to guarantee that the cancellation takes place at all orders.

Supersymmetry

Such a symmetry is called a supersymmetry:

$$Q|\mathrm{boson}\rangle = |\mathrm{fermion}\rangle \qquad Q|\mathrm{fermion}\rangle = |\mathrm{boson}\rangle$$

The symmetry generator Q (and its hermitian conjugate $Q^\dagger$) carry spin 1/2: it is a space-time symmetry.

The form of possible supersymmetry algebras is strongly constrained on the basis of very general theorems in field theory. For example, it is impossible with ordinary symmetry generators (elements of a commutator algebra).

Supersymmetry

There is essentially one possibility:

$$\{Q, Q^\dagger\} = P^\mu$$
$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$$
$$[P^\mu, Q] = [P^\mu, Q^\dagger] = 0$$

(more on this later). Further specifications:

- $Q, Q^\dagger$ **transform as spinors under the Lorentz group**
- $Q, Q^\dagger$ **commute with gauge symmetry generators.**

In principle, we may have more than one Q**:** Q^i, $i = 1, \ldots, N$ **(extended supersymmetry).**

Supersymmetry

A few basic properties of a supersymmetric theory can already be recognized:

- **particles in the same supersymmetric multiplet (which we will call a supermultiplet) have equal masses and equal gauge transformation properties (electric charge, weak isospin and color)**
- **within the same supermultiplet, there is an equal number of bosonic and fermionic degrees of freedom (a proof on the next slide)**

Supersymmetry

Fermionic fields: a reminder

We use the Weyl representation of the Dirac matrices:

$$\gamma^\mu = \begin{bmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{bmatrix} \qquad \sigma^\mu = (\sigma^0, \vec{\sigma}) \qquad \bar{\sigma}^\mu = (\sigma^0, -\vec{\sigma}) \qquad \gamma_5 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

where $\vec{\sigma}$ are the three Pauli matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Supersymmetry

The familiar lagrangian for a free, massive Dirac spinor is

$$\mathcal{L} = \bar{\psi}\,(i\partial\!\!\!/ - m)\,\psi \qquad (\partial\!\!\!/ = \gamma^\mu \partial_\mu)$$

$$\psi = \begin{pmatrix} \xi_L \\ \xi_R \end{pmatrix} \quad \psi_L = \begin{pmatrix} \xi_L \\ 0 \end{pmatrix} = \frac{1}{2}(1-\gamma_5)\psi \quad \psi_R = \begin{pmatrix} 0 \\ \xi_R \end{pmatrix} = \frac{1}{2}(1+\gamma_5)\psi$$

Four-component Dirac spinors realize a reducible representation of the Lorentz group: the two-component spinors ξ_L, ξ_R transform independently under Lorentz transformations.

In terms of ξ_L, ξ_R we have

$$\mathcal{L} = i\xi_L^\dagger\,\bar{\sigma}^\mu \partial_\mu\,\xi_L + i\xi_R^\dagger\,\sigma^\mu\,\partial_\mu \xi_R - m(\xi_L^\dagger \xi_R + \xi_R^\dagger \xi_L)$$

Supersymmetry

It can be shown that the transformation law of the two-component spinor ξ_R under Lorentz transformations are the same as those of $\epsilon\xi_L^*$, where ϵ is the antisymmetric matrix

$$\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} .$$

It follows that any Dirac spinor may always be written in terms of two left-handed Weyl spinors ξ, χ as

$$\psi = \begin{pmatrix} \xi \\ -\epsilon\chi^* \end{pmatrix}$$

(the minus sign on the second Weyl spinor is conventional).

Four-component vs. Two-component fermions

- A Dirac Spinor is a four component object whose components are

$$\psi_D = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}; \qquad \psi_D^C = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} \tag{8}$$

- A Majorana Spinor is a four component object whose components are

$$\psi_M = \begin{pmatrix} \chi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}; \qquad \psi_M^C = \psi_M \tag{9}$$

- Gamma Matrices

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}; \qquad \gamma^5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix} \tag{10}$$

- Observe that $\psi_{D,L} = \chi$; $\psi_{D,R} = \bar{\psi}$

- Usual Dirac contractions may be then expressed in terms of two component contractions.

$$\bar{\psi}_D = (\psi^\alpha \quad \bar{\chi}_{\dot{\alpha}}) \tag{11}$$

- For instance,

$$\bar{\psi}_D\,\psi_D = \psi\chi + h.c.; \tag{12}$$

$$\bar{\psi}_D\gamma^\mu\psi_D = \psi\bar{\sigma}^\mu\bar{\psi} + \bar{\chi}\sigma^\mu\chi \quad = \quad -\bar{\psi}\sigma^\mu\psi + \bar{\chi}\sigma^\mu\chi \tag{13}$$

Observe that Majorana particles lead to vanishing vector currents. Therefore, they must be neutral under electromagnetic interactions. Chiral currents don't vanish, $\bar{\psi}_D\gamma^\mu\gamma_5\psi_D = -\bar{\psi}\sigma^\mu\psi - \bar{\chi}\sigma^\mu\chi$. They may couple to the Z-boson.

- Other relations may be found in the literature.

Supersymmetry

The free lagrangian for a massive Dirac spinor is

$$\begin{aligned}\mathcal{L} = \bar{\psi}\,(i\partial\!\!\!/ - m)\,\psi &= i\chi^T\,\epsilon\sigma^\mu\epsilon\,\partial_\mu\chi^\dagger + i\xi^\dagger\,\bar{\sigma}^\mu\partial_\mu\xi - m(\chi^T\epsilon\xi - \xi^\dagger\epsilon\chi^*) \\ &= i\chi^\dagger\,\bar{\sigma}^\mu\,\partial_\mu\chi + i\xi^\dagger\,\bar{\sigma}^\mu\partial_\mu\xi - m(\chi^T\epsilon\xi - \xi^\dagger\epsilon\chi^*)\end{aligned}$$

Note that

$$\chi^T\epsilon\xi = \xi^T\epsilon\chi$$

because of the anticommutation properties of fermion fields, and that

$$(\chi^T\epsilon\xi)^\dagger = \xi^\dagger\epsilon^T\chi^* = -\xi^\dagger\epsilon\chi^*$$

The shorthand notation

$$\chi\xi = \xi\chi = i\chi^T\epsilon\xi \qquad \xi^\dagger\chi^\dagger = \chi^\dagger\xi^\dagger = -\xi^\dagger\epsilon\chi^*$$

is often used.

Supersymmetry

Any theory involving spin-1/2 particles may be written as a collection of left-handed Weyl spinors ψ_i.

Left-handed fermions in the Standard Model:

$$u_i, d_i, e_i, \nu_i$$

$$\bar{u}_i, \bar{d}_i, \bar{e}_i$$

where $i = 1, 2, 3$ is a generation index. For example, the Dirac spinor for the up quark is written in terms of two left-handed Weyl spinors u and $\bar{u}$ as

$$\psi_u \equiv \begin{pmatrix} u_L \\ u_R \end{pmatrix} = \begin{pmatrix} u \\ -\epsilon \bar{u}^* \end{pmatrix}$$

(NB: the bar on $\bar{u}$ here has no special meaning, it is just part of the name)

Supersymmetry

The Weyl notation is convenient, since left- and right-handed components of Dirac fermions, which behave differently in weak interactions, are treated separately. Furthermore, the simplest (irreducible) representations of supersymmetry contain Weyl fermions.

The simplest supermultiplet candidate:

a Weyl fermion ψ and two real (or one complex) scalar ϕ_1, ϕ_2.

This is called a chiral or matter supermultiplet.

Supersymmetry

Next-to-simplest possibility: a supermultiplet that contains a massless gauge vector boson A^a_μ. Two bosonic degrees of freedom, so its partner must be again a Weyl fermion, called a *gaugino*, λ^a (it cannot be a spin-3/2 field: we want a renormalizable theory).

Gauge bosons belong to the adjoint representation of the gauge group, so the same is true for their supersymmetric partners. Thus,

gauginos are not chiral fermions

because the adjoint representation is equivalent to its conjugate.

The pair A^a_μ, λ^a is called a gauge supermultiplet.

SUSY SM

The particle content of a supersymmetric SM

- No supermultiplet can be formed out of standard particles (e.g. γ, ν).
- Matter fermions must belong to chiral supermultiplets, because left and right fermions transform differently under the weak gauge group.
- Gauge bosons must go into gauge supermultiplets.
- At least two Higgs doublets, with their fermionic partners: cancellation of the axial anomaly.

SUSY SM

spin 0	spin 1/2	spin 1	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$\tilde{u}_L, \tilde{d}_L$	u_L, d_L		$\mathbf{3}$	$\mathbf{2}$	$+\frac{1}{3}$
$\tilde{u}_R$	u_R		$\mathbf{3}$	$\mathbf{1}$	$+\frac{4}{3}$
$\tilde{d}_R$	d_R		$\mathbf{3}$	$\mathbf{1}$	$-\frac{2}{3}$
$\tilde{\nu}, \tilde{e}_L$	ν, e_L		$\mathbf{1}$	$\mathbf{2}$	-1
$\tilde{e}_R$	e_R		$\mathbf{1}$	$\mathbf{1}$	-2
H_u^+, H_u^0	$\tilde{h}_u^+, \tilde{h}_u^0$		$\mathbf{1}$	$\mathbf{2}$	$+1$
H_d^0, H_d^-	$\tilde{h}_d^0, \tilde{h}_d^-$		$\mathbf{1}$	$\mathbf{2}$	-1
	$\tilde{g}$	g	$\mathbf{8}$	$\mathbf{1}$	0
	$\tilde{w}^\pm, \tilde{w}^0$	$W^\pm, W^0$	$\mathbf{1}$	$\mathbf{3}$	0
	$\tilde{b}^0$	B^0	$\mathbf{1}$	$\mathbf{1}$	0

SUSY SM

Supersymmetric partners of standard particles (e.g. a scalar electron) with the same masses would have been detected in experiments. Since none of them has been observed so far, we must conclude that

supersymmetry must be broken

in a realistic theory. Recall our formula for scalar mass corrections:

$$(\Delta m^2)_{a+b} = \frac{\lambda_S - |\lambda_f|^2}{8\pi^2}\,\Lambda^2 + \dots$$

Supersymmetry forces $\lambda_S = |\lambda_f|^2$ to all orders in perturbation theory, so that quadratic divergences are systematically cancelled. This feature must be preserved in the broken theory:

$$\mathcal{L} = \mathcal{L}_{\text{supersymmetric}} + \mathcal{L}_{\text{soft}}$$

where $\mathcal{L}_{\text{soft}}$ only contains mass terms and couplings with positive mass dimension.

SUSY SM

Supersymmetric partners of ordinary particles are so heavy that they have escaped detection so far.

Is there a reason for that?

All ordinary particles, including the W, Z bosons and the top quark, would be massless in the absence of spontaneous breaking of the electroweak gauge symmetry.

The contrary is true for their partners: scalar masses are always allowed by gauge symmetries, and gaugino can be massive because they belong to a real representation of the gauge group.

The only exception is the Higgs boson.

SUSY SM

A rough estimate of superparticle masses

Call $m_{\rm soft}$ the largest mass scale present in $\mathcal{L}_{\rm soft}$. The corrections to m^2 arising from $\mathcal{L}_{\rm soft}$ must vanish as $m_{\rm soft} \to 0$, so they cannot grow as Λ^2. Corrections proportional to $m_{\rm soft}\Lambda$ are also forbidden (UV divergences are either quadratic or logarithmic). Therefore

$$\Delta m^2 \sim \lambda\, m_{\rm soft}^2 \log \frac{\Lambda}{m_{\rm soft}}$$

(λ a generic coupling). Furthermore,

$$m_F^2 - m_B^2 \sim m_{\rm soft}^2$$

within a supermultiplet. It follows ($\lambda \sim 1, \Lambda \sim M_P$) that

$$m_{\rm soft} \lesssim 1 \text{ TeV}$$

in order to get the correct value of the Higgs v.e.v.