Beyond the Standard Model

Lecture 7

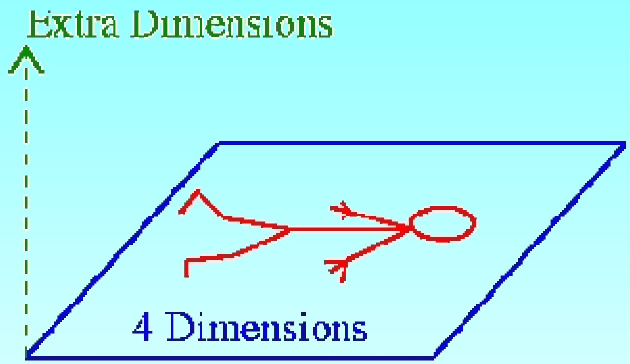
Leandar Litov
University of Sofia

EXTRA DIMENSIONS

Kaluza – Klein compactification

EXTRA DIMENSIONS

- It is a fact of life that the observed world lives in 4=1+3 dimensions.
- However one can easily conceive the existence of extra dimensions:

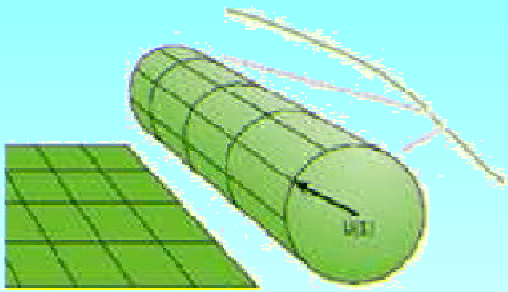


- In 1921 T. Kaluza proposed the idea of a 5-th physical dimension. This idea was further elaborated by O. Klein in 1926.
- Their motivation was to show that in 5 dimensions gravity and electromagnetism could be unified into a single theory.
- The idea is that we do not observe usually the extra dimensions because we need an enormous energy to jump into them.

- This idea was retaken in the 70's and 80's in the context of supergravity theories.
- The maximal $D = 4$ supergravity theory, $N = 8$ supergravity, was considered as a serious candidate for a unified theory of all interactions.
- It may be obtained starting from $N = 1$ supergravity in $D = 11$ upon compactification, a la Kaluza-Klein.
- The size of the extra dimensions was necessarily of the order of $1/M_{Planck}$, extremely small.
- Unfortunately in the early 80's it was shown that this scheme could not contain the SM[a].
- Either the theory was non-chiral and/or the gauge group was too small to contain $SU(3) \times SU(2) \times U(1)$.
- This has been retaken in the last 15 years for the following two main reasons:
  - In the context of string theory one can obtain chirality and a gauge group big enough to contain the SM (1985).
  - It has been realized that the size of extra dimensions could be much larger than previously thought. So low that their effects could perhaps be observed, e.g., at LHC (1998).

[a]Witten (1981)

Geometric idea for unifying electromagnetism and gravity; Compactified 5D space:



- The noncompact theory has full 5D Lorentz invariance - graviton g^{MN} where $M, N = 0...4$
- Compact theory has 4D Lorentz invariance plus U(1)

TOY EXAMPLE: A SCALAR IN 5 DIM.

- Consider a free scalar $\Phi(x_\mu, y)$ in $D = 5$. The Lagrangian has the simple form:

$$L_\Phi = -\frac{1}{2}\partial_A \Phi \partial^A \Phi \ , \ \ A = 0,1,2,3,4$$

- We now assume that the 5-th coordinate y is curled into a circle of radius R. Thus the geometry is $M_4 \times S^1$. we should identify:

$$\Phi(x_\mu, y) = \Phi(x_\mu, y + 2\pi R)$$

- Then one can expand in harmonics on the circle:

$$\Phi(x_\mu, y) = \sum_{n=-\infty}^{+\infty} \Phi(x_\mu)_n e^{iny/R}$$

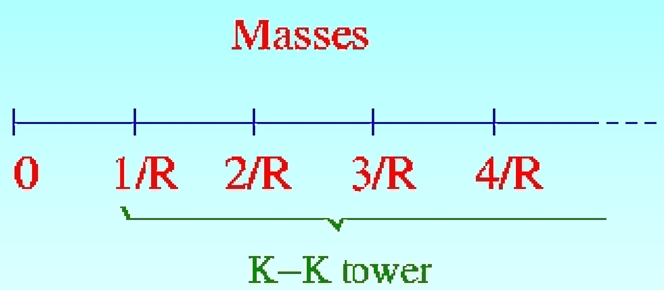
- Substituting and redefining $\phi_n = \sqrt{2\pi R}\Phi_n$ one obtains:

$$S_\Phi = \int d^4x \left(-\frac{1}{2}\partial_\mu \phi_0 \partial^\mu \phi_0\right)$$

$$- \int d^4x \sum_{n=1}^{+\infty} \left(\partial_\mu \phi_n \partial^\mu \phi_n^* + \frac{n^2}{R^2}\phi_n \phi_n^*\right)$$

- In summary we get:
- – A massless scalar ϕ_0
  – An infinite tower of massive scalars with mass

$$m_n^2 = \frac{n^2}{R^2} \quad ; \quad n = 1, 2, 3..$$



- For energies $\ll 1/R$ only the zero mode is visible.
- Thus if $1/R \gg$ 1 Tev one understands why we have not seen extra dimensions yet.

Things become much more interesting if we consider gravity, instead of just a scalar field.

GRAVITY AND A 5-TH DIMENSION

- Recall the action of Einstein gravity in $D = 4$:

$$S_4 = \frac{M^2_{Planck}}{2} \int d^4x \sqrt{g} R_4(g)$$

where $g = det(g_{\mu\nu})$ and R_4 is the scalar curvature.

- Consider now the action for gravity in 5 dimensions:

$$S_5 = \frac{M^3_5}{2} \int d^4x dy \sqrt{G} R_5$$

where $G = det(G_{AB})$, with A,B=0,1,..,4 and R_5 is the scalar curvature in 5 dimensions.

- Let us compactify again the 5-th dimension in a circle with radius R. Then we can expand the metric in harmonics:

$$G_{AB}(x_\mu, y) = \sum_{n=-\infty}^{+\infty} G_{AB}(x_\mu)_n e^{iny/R}$$

- As in the case of scalar field there are massless particles and an infinite tower of massive gravitons.
- Let us label the massless components of the 5-dim. metric in the following way:

$$G^0_{MN} = \left(\begin{array}{c|c} g_{\mu\nu}(x) e^{\sigma/\sqrt{3}} + A_\mu A_\nu & e^{-2\sigma/\sqrt{3}} A_\mu(x) \\ \hline e^{-2\sigma/\sqrt{3}} A_\mu & e^{-2\sigma(x)/\sqrt{3}} \end{array} \right)$$

- This shows explicitly that the metric G_{MN} in $D=5$ gives rise to:
  - a massless $D=4$ graviton $g_{\mu\nu}$
  - a gauge boson A_μ
  - a scalar *radion* σ
- $D=4$ gravity and a gauge *photon* are unified in $D=5$!
- This (kaluza-Klein) idea was the first attempt for the unification of gravity and electromagnetism.

- Substituting in 5-D action one gets the action for the zero mass fields

$$S^0_{KK} = M_5^3 \pi R \int d^4x \sqrt{g} \left(R_4(g) - \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{4e^{\sqrt{3}\sigma}} F^2_{\mu\nu} \right)$$

- Comparing with the $D-4$ gravity action $\frac{1}{2} M^2_{Planck} \int d^4x \sqrt{g} R_4$ one observes:

$$M^2_{Planck} - M_5^3 2\pi R$$

- On the other hand we know:

$$M_{Planck} = \frac{1}{\sqrt{8\pi} G^{1/2}_{Newton}} = 1.2 \times 10^{19}\, GeV$$

- These equations seem to indicate that the $D = 5$ fundamental scale M_5 could be arbitrarily low (as low e.g. as the electroweak scale !) if we make the radius R sufficiently large.
- This is NOT POSSIBLE for several reasons. In particular recall that matter fields have KK replicas with masses $m_n^2 = n^2/R^2$. We should have observed the KK replicas of SM particles if R was so large!
- BRANE WORLD idea will change this fact!

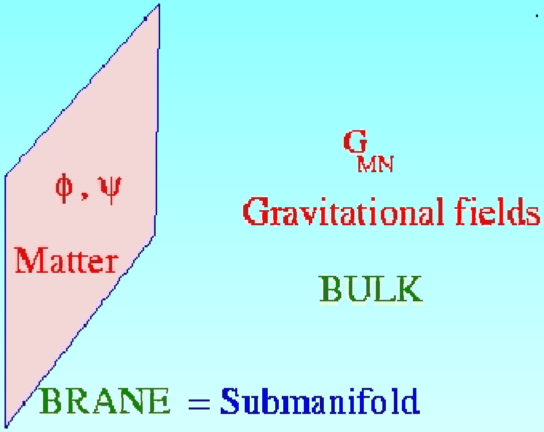
PROBLEMS WITH CHIRAL FERMIONS

- Up to now we have only talked about bosons in extra dimensions. What about fermions, needed to accommodate quarks and leptons.
- We already remarked that fermion chirality is a very fundamental property of the SM. The fundamental building blocks in the SM are Weyl fermions, i.e., bispinors, NOT Dirac 4-spinors.
- $D = 4$ Chiral fermions are difficult to get from theories in extra dimensions. The reason is that spinors in $D > 4$ always contain upon dimensional reduction Dirac 4-spinors, never Weyl 2-spinors.
- One way to get chiral fermions, widely used in heterotic string theory, is the addition of explicit (no KK) gauge fields. Backgrounds for those fields in extra dimensions modify Dirac eq. in extra dimensions and allow for massless Weyl fermions.
- On the other hand our view about chirality in extra dimensions is one of the questions which has changed with the advent of the brane world idea.

Extra Dimensions and Brane Warld

BRANEWORLD IDEA

- The idea is very simple. In certain systems non-gravitational fields, i.e. SM fields, may be forced to live in a sub-manifold of the full space.
- That sub-manifold is called a BRANE.



- The BRANE contains the observed 4-dimensions.
- It is important to realize that those matter fields are forced to live in a smaller number of dimensions by some dynamical reason, not by a compactification process.
- On the other hand gravitational interactions occupy the full space, the BULK of space.

- There are field theory configurations in which that localization process takes places naturally [a]. But that localization of matter on a subspace is particularly natural in open string theory.
- As we will see later, in string theory there are objects called D-branes which have precisely the property of localizing matter and gauge fields on a a sub-manifold of the full 10-dimensional space.
- There are branes of different dimensionality. Conventionally a n-BRANE has $n+1$ dimensions.
- Thus one can assume e that the observed (non-gravitational) world is inside a 3-brane.
- Note than then the action of the system will split into a 4-dimensional piece (for the 3-BRANE) and a D-dimensional piece (the BULK):

$$S = S_{brane} + S_{bulk} =$$
$$\int d^4x L_{brane} + \int d^Dx L_{bulk}$$

[a] Rubakov,Shaposnikov (1983)

- To get the usual 4-dimensional gravity one should compactify the $D-4$ extra dimensions, as we did in the 5-dimensional example.
- However, since matter fields are 4-dimensional to start with, the compactification does not affect them at all.
- In particular, in the case of a 3-brane, MATTER FIELDS DO NOT HAVE KK MASSIVE COPIES!!.
- This is the key property why in the brane scenario extra dimensions could be around the corner (i.e. LHC).
- There are at least two interesting consequences of the braneworld scenario:
  - It is easy to have chiral fermions and extra dimensions simultaneously, since fermions live to start with in 4 dimensions.
  - One can safely have very large compact radii R without a tower of KK replicas of the SM particles becoming light, since SM fields do not have KK towers.
- These are the two general ideas underlying the popular large extra dimensions scenario which we now briefly review.

THE LARGE EXTRA DIMENSIONS SCENARIO

- This was proposed in 1998 by Arkani-Hamed, Dimopoulos and Dvali as an alternative to understand the hierarchy problem.
- Consider a $D = (4+n)$ dimensional space with matter fields in a 3-brane.
- The action has the general form:

$$S = \frac{M_*^{2+n}}{2} \int d^4x \int d^n y \sqrt{G} R_{(4+n)} + \int d^4x \sqrt{g} L_{SM}$$

- Assuming here for simplicity all n compact dimensions with the same size R, the gravity action in 4-dimension reads:

$$\longrightarrow \frac{M_*^{2+n}(2\pi R)^n}{2} \int d^4x \sqrt{g} R_4$$

- Thus one gets for the Planck mass:

$$M_{Planck}^2 = M_*^{2+n}(2\pi R)^n$$

EXTRA DIMENSIONS AROUND THE CORNER

$$M_{Planck}^2 = M_*^{2+n}(2\pi R)^n$$

- This equation shows that the fundamental scale M_* may be as small as we wish as long as the size R of extra dimensions is sufficiently large.
- In particular one can have M_* = 1Tev with radii:

n	1	2	3	...	7
R	10^8 Km	0.1 mm	$10\ ^{6}$ mm	...	$10\ ^{12}$ mm
R^{-1}	10^{-18} eV	10^{-3} eV	100 eV	...	100 MeV

- Note that in this scenario there is no hierarchy problem in the traditional sense: since both electroweak M_W and fundamental gravitational scale M_* are of order 1 TeV.
- In fact the new version of the hierarchy problem is to understand why the extra dimension scale $1/R$ is so small compared to the fundamental scale M_*.

- The Eöt-Wash (2002) group obtains for n=2 [a]:

$$\lambda < 0.15mm \ (95\%CL)$$

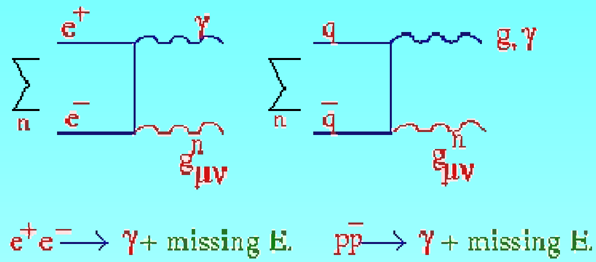
- This implies $M_* \geq 1TeV$, for n=2 large extra dimensions.
- The case with only one large extra dimension $n = 1$, $R = 10^8 Km$, is excluded by astronomical observations.
- There are also direct bounds from accelerator limits which we briefly describe now.

---

[a] Acelberger et al. (2002)

PRODUCING LED GRAVITONS AT ACCELERATORS

- Extra dimensional gravitons $g^{n}_{\mu\nu}$ may be directly produced at colliders along with a photon or a jet [a] :



- The rate for the production of *an individual* graviton is suppressed by $1/M^2_{Planck}$. But there are so many contributing that the inclusive cross section is important:

$$\sigma \simeq \frac{E^n_{C.M.}}{M_*^{n+2}}$$

- From LEP and Tevatron one gets limits [b]:

n	2	3	4	5	6
M_*(TeV)	0.57	0.36	0.26	0.19	0.16

In LHC one expects to test up to $M_* \simeq 2-3$ TeV for $n = 4-3$.

[a]Giudice et al;Mirabelli et al.(1999)

[b]Giudice,Strumia (2003)

ASTROPHYSICS CONSTRAINTS

- In fact astrophysics limits turn out to be stronger than accelerator ones.
- There are several specific sources for the constraints
  - From supernova cooling (SN1987A). From KK graviton emission during explosion of supernovae. It should not deplete too much the observed neutrino flux from SN1987A.
  - Production of diffuse γ-rays from decays of the KK gravitons trapped in the neutron star halo.
  - The photon flux from KK graviton decay should not overheat the neutron star surface .
- One finds very strong lower bounds on M_* (see Hannestad and Raffelt (2003)):

n	2	3	4
SN cooling (SN1987A)	8.9 TeV	660 GeV	10 GeV
Neutron Star heat excess	700 TeV	25.5 TeV	2.77 TeV
Diffuse γ-rays	38.6 TeV	2.65 TeV	430 GeV

It seems LHC unlikely to be able to see (this kind of) extra dimensions with $M_* \simeq 1$ TeV unless $n \geq 4$.

SM particles living in extra dimensions

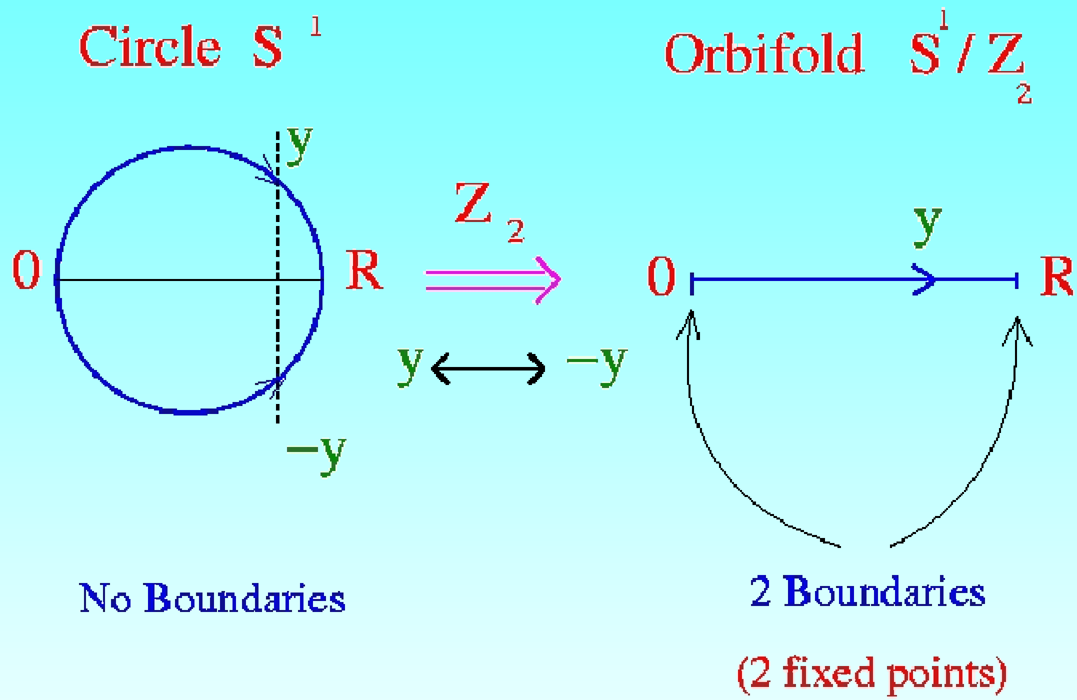
SM PARTICLES LIVING IN EXTRA DIMENSIONS

OF SIZE 1/TeV

- Up to now we have considered SM particles living in a 3-brane and hence with no massive KK copies
- More generally one could consider the possibility of SM living in $D = 4 + \delta$ with δ dimensions of size e.g 1/TeV.
- If that is the case the phenomenology is rather different compared to the previous case. Thus, e.g., one could produce KK copies of SM particles at colliders.
- This is still compatible with having other *different* extra dimensions very large to explain why $M_{Planck} \gg M_W$.
- Many models of this type have been constructed which depend on whether quarks and/or leptons and/or gauge bosons live in extra dimensions or are localized on a 3-brane .
- In order to get chiral fermions one compactifies on an ORBIFOLD [a] instead of a circle or a torus.

[a] Dixon, Harvey, Vafa, Witten (1985)

THE SIMPLEST ORBIFOLD EXAMPLE



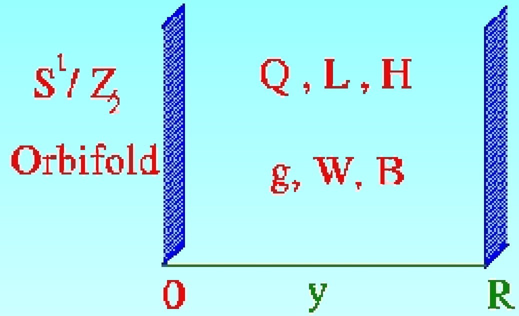
- For us the important point is that it provides with a method to obtain chiral fermions starting with a non-chiral $D > 4$ theory.
- Indeed, the 5-dimensional fields are forced to be invariant under the Z_2 operation.
- The Z_2 projection halves the number of $D = 4$ fermion degrees of freedom and convert massless Dirac into Weyl spinors.

MODELS WITH 1 TEV EXTRA DIMENSIONS

- There is an enoooormous literature with different possibilities.

Example

- One starts with the usual fields of the SM in $D=5$ and compactify in the orbifold S^1/Z_2 [a]



All SM particles live in D=5
(none on the boundaries)

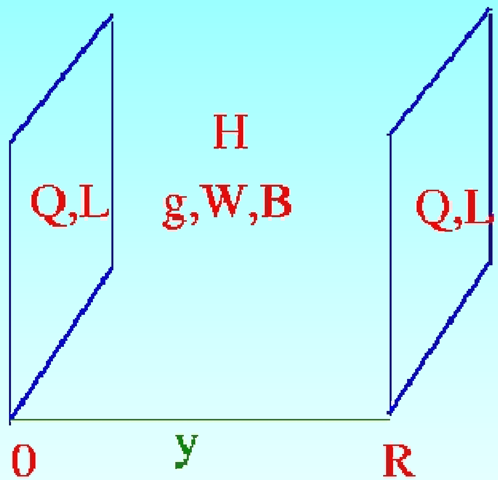
- All SM particles have KK replicas with masses of order n/R. There may be more than one 1/TeV extra dim.
- There is a residual Z_2 symmetry such that KK replicas of SM fields have to be pair-produced at colliders. Lightest KK modes are stable.

[a] Appelquist, Cheng, Dobrescu (2001).

- From production of KK gluons and quarks one expects to be sensitive to $1/R \simeq$ 300-500 GeV at Tevatron, up to 3 TeV at LHC.

Other Example

- One can have gauge bosons and Higgs in the bulk and quarks and leptons on some 3-branes on the boundaries. For example [b]



Higgs and gauge bosons in bulk

Quarks and leptons in branes

- If some standard model fields live on 3-branes on the boundaries, there is no residual Z_2 symmetry left.

[b] Pomarol, Quiros (1998)

- The possibilities are multiple and are not easy to summarize.
- Some possibilities considered:
  - Orbifold S^1/Z_2 models with $N = 2$ supersymmetry on bulk and $N = 1$ supersymmetry on branes[c]
  - Orbifold $Z_2 \times Z_2$ models with different $N = 1$ SUSY's on the two branes but no SUSY globally[d].
  - Grand unified SU(5) [e] or SO(10) theories in 5 and 6 dimensions. In this case the doublet-triplet problem can easily be solved. The compactification scale is of order the unification mass.
  - Etc....

---

[c]Pomarol and Quiros; Antoniadis et al.; Delgado et al.(1998)
[d]Barbieri, Hall, Nomura (2000)
[e]Kawamura; Hall and Nomura, (2001)

SOME QUESTIONS TO ADDRESS

1) B- and L-number violation

- In a theory with fundamental scale $M_* = 1$ TeV there is no obvious symmetry forbidding operators of the form

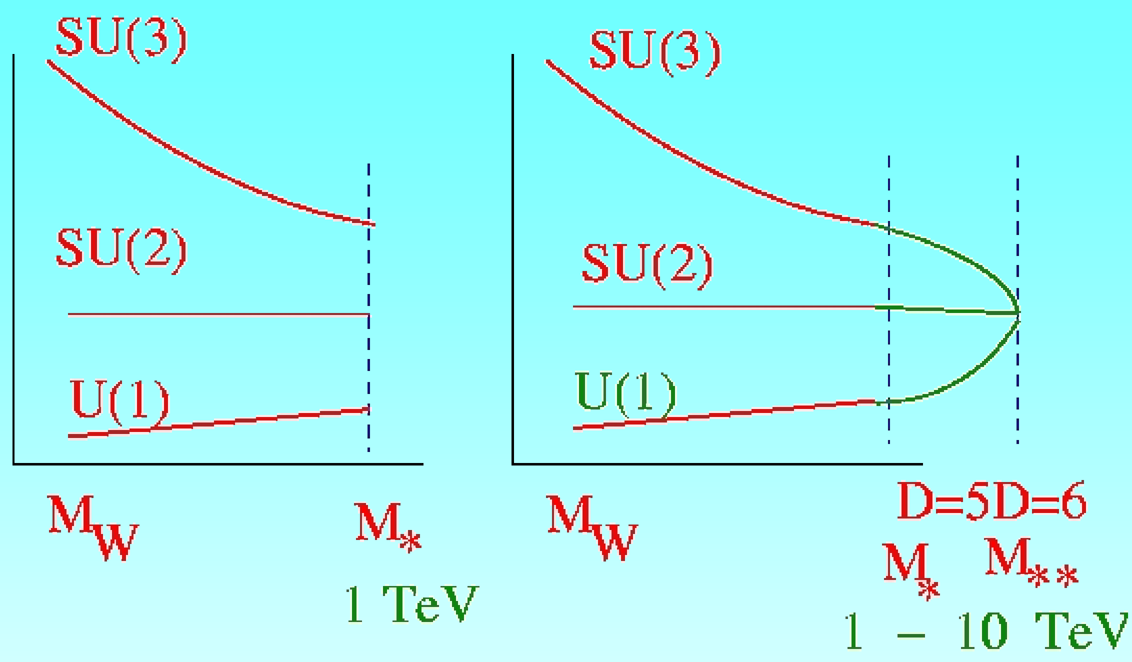
$$\frac{1}{M_*^2}(qqqL)$$

- They induce proton decay at a rate of a few minutes...
- Such operators are highly suppressed in e.g. GUT's because the relevant scale is very large $M_X \simeq 10^{16}$ GeV.
- Thus in LED scenarios there should be some symmetry or mechanism guaranteeing sufficient proton stability.
- There are however elegant mechanisms in e.g. string theory which guarantee sufficient stability.
- In particular Baryon and Lepton $U(1)$ symmetries may be gauged guaranteeing perturbative stability.

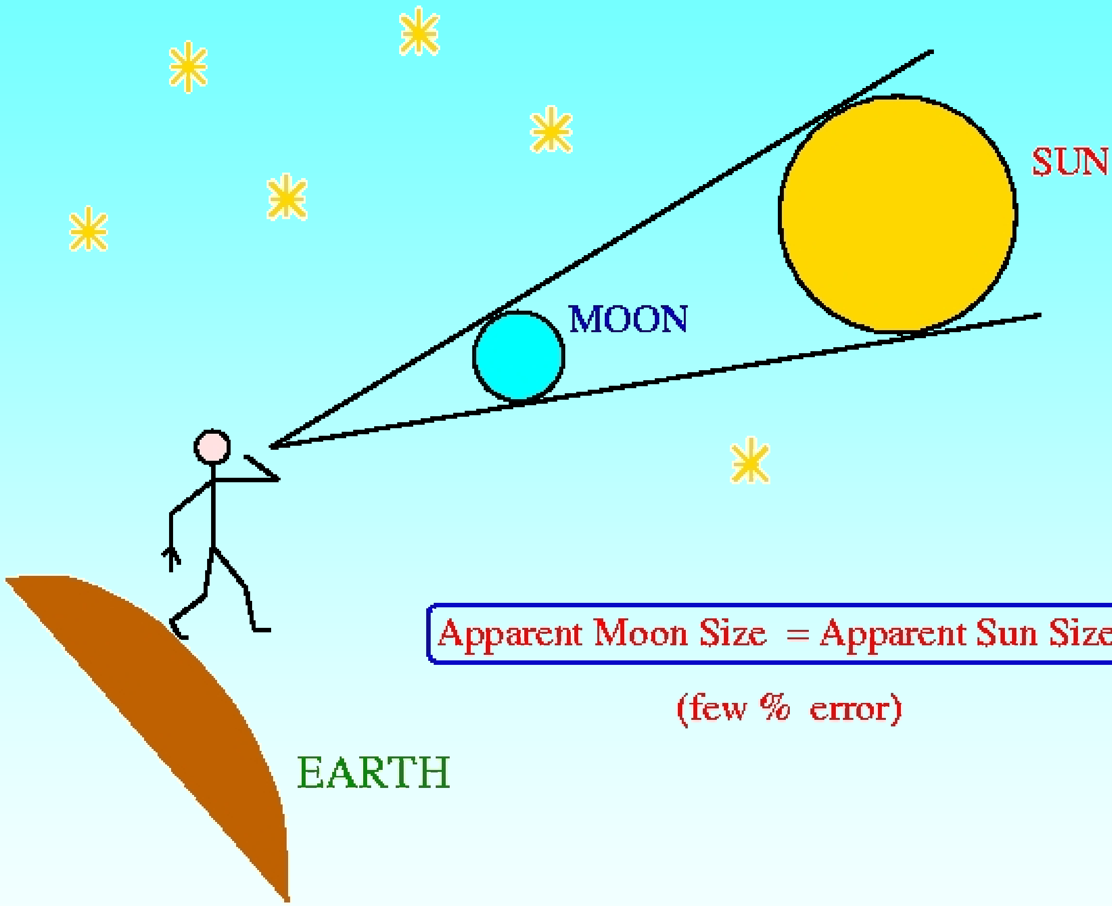
2) Gauge coupling unification

- One of the best aspects of SUSY+GUT's is the beautiful unification of gauge coupling constants.
- That success relies on 3 points: MSSM particle content, GUT coupling relationships and assumption of a 'big desert' in between M_W and M_X .
- In the LED scenarios the desert disappears and couplings in general do not seem to unify at such a low fundamental scale like $M_* = 1TeV$!!
- It has been suggested [a] that there may be *accelerated unification* if some SM particles live in extra dimensions of size $\sim 1/M_*$.
- However this seems to require certain degree of fine-tuning.

[a] Dines, Dudas, Gherghetta, (1999)



- Alternatively apparent gauge coupling unification could be a coincidence.
- In nature there are already examples in which these coincidences take place. For example, the unification of the apparent size of the sun and the moon!:

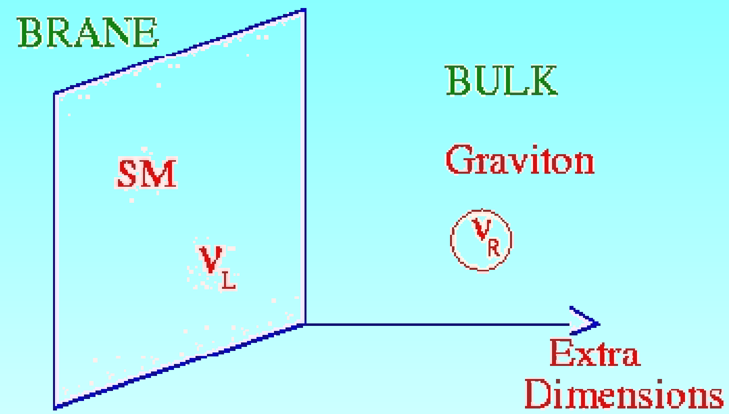


Apparent Moon Size = Apparent Sun Size

(few % error)

3) Smallness of neutrino masses

- There is a natural mechanism to understand the smallness of neutrino masses in LED theories [a].



- Since right-handed neutrinos (like also gravitons) are singlet under SM interactions they could perhaps live in the bulk of space.
- In such a case Yukawa couplings giving Dirac masses to neutrinos are suppressed by factors of $1/M^2_{Planck}$:

[a] Dienes, Dudas, Gherghetta (1998); Dvali, Smirnov (1999)

- Consider e.g. three 4-component spinors in 5 dimensions (Ψ_L^a, Ψ_R^a) , $a = 1, 2, 3$ with a usual $D = 4$ Yukawa coupling to the left-handed neutrinos ν_L^a and the Higgs scalar H:

$$S_\nu = \int d^4x \int_0^R dy\, \overline{\Psi}^a \Gamma_M \partial^M \Psi^a$$
$$+ \int d^4x \left(i\overline{\nu}_L^a \gamma_\mu \partial^\mu \nu_L^a + Y_{ab} H \overline{\nu}_L^a \Psi_R^b + h.c. \right)$$

- We redefine $\nu_R = \Psi_R^b \sqrt{R} = \Psi_R^b (M_{Planck}/M_*^{3/2})$ and $Y_{ab} = (M_*)^{-1/2} h_{ab}$. After the Higgs H gets a vev$= v$ one gets minute Dirac neutrino masses:

$$L_\nu = v\left(\frac{M_*}{M_{Planck}}\right) h_{ab} \overline{\nu}_L^a \Psi_R^b + h.c.$$

- Thus one has typically $m_\nu \simeq (M_W M_*)/M_{Planck}$, a sort of sea-saw result.

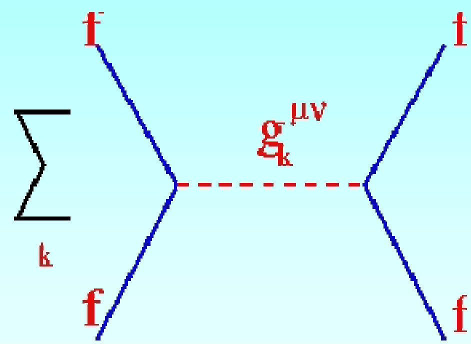
- Ψ_R^b lives in 5 dimensions and has KK replicas with masses $\simeq 1/R$. Thus upon diagonalization the physical ν_L's mix with the infinite tower of KK right-handed neutrinos.

- These behave like an infinite tower of *sterile neutrinos*.
- In fact active neutrinos can in principle oscillate into these extra dimensional neutrinos.
- However SNO + Kamiokande results imply a very small contribution from sterile neutrinos. Thus the observed oscillations cannot be due to oscillations into extra-dimensional neutrinos.
- In fact one can put bounds on the largest extra dimension size by requiring not too large sterile contribution.

Detecting flat extra dimensions

DEVIATIONS FROM NEWTON's LAW

- Recall that graviton (unlike usual matter!!) has Kaluza-Klein replicas with masses $m_n^2 = n^2/R^2$.
- Those KK gravitons are very light for large radii (see previous table). They have not been observed because their couplings to usual matter are suppressed by powers of $1/M_{Planck}$.
- Still they can be exchanged and give rise to deviations from Newton's law :



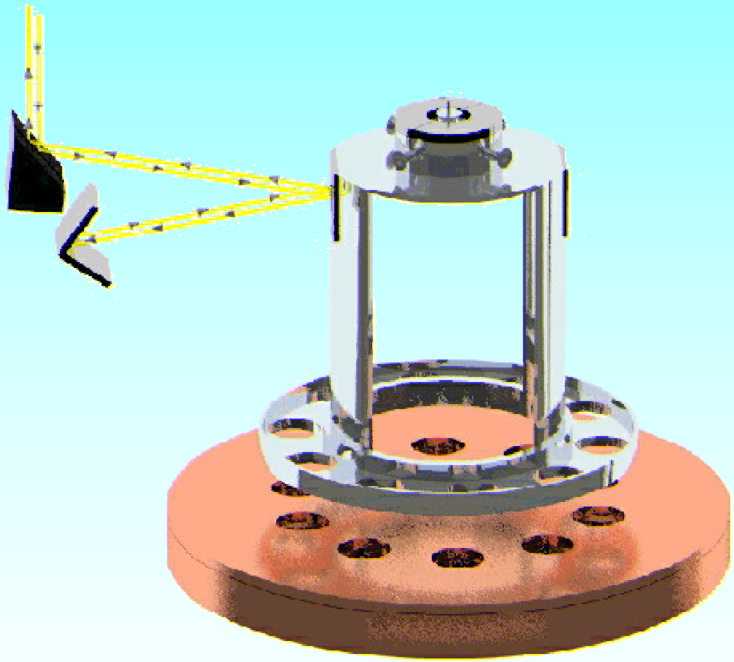
- Deviations experimentally parametrized by

$$V(r) = -\frac{G_N m}{r}\left(1 + 2\alpha e^{-\frac{r}{\lambda}}\right)$$

- LED models give this kind of deviation with α = 2n, $\lambda = 2\pi R$.

Detecting flat extra dimensions

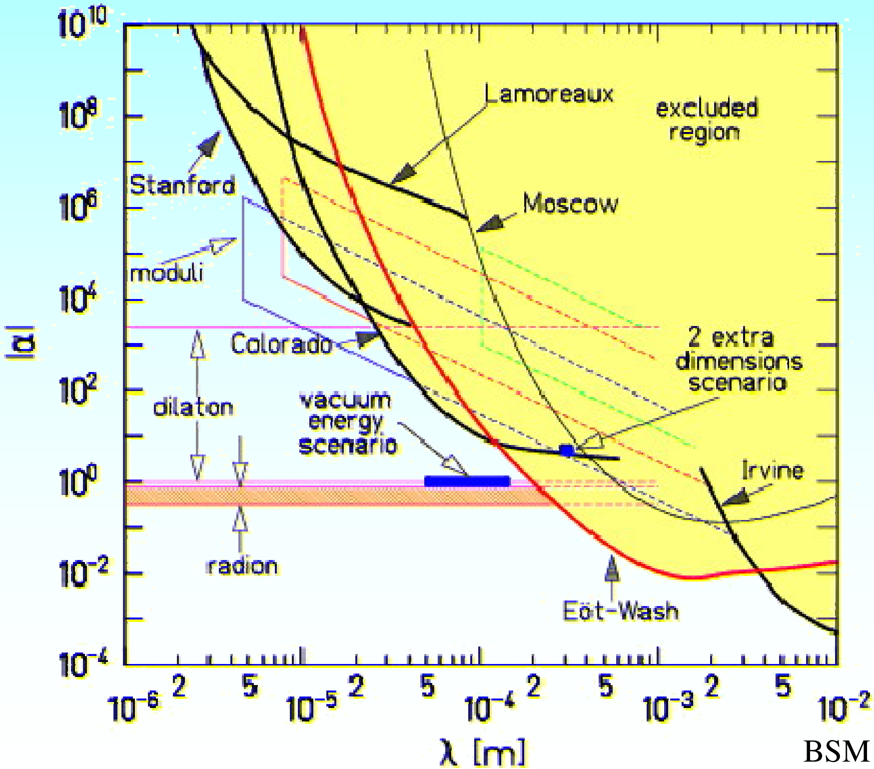
Detection1: Eotvos (torsion balance) experiments - e.g. C.Hoyle et al, hep-ph/0011014, hep-ph/0405262 find $R_{2-extra} < 0.13mm$ at 95% c.l.



Usually present ISL violation as a Yukawa interaction

$$V(r) = -\frac{Gm_1m_2}{r}(1+\alpha e^{-r/\lambda})F^{\mu\nu}F_{\mu\nu}$$

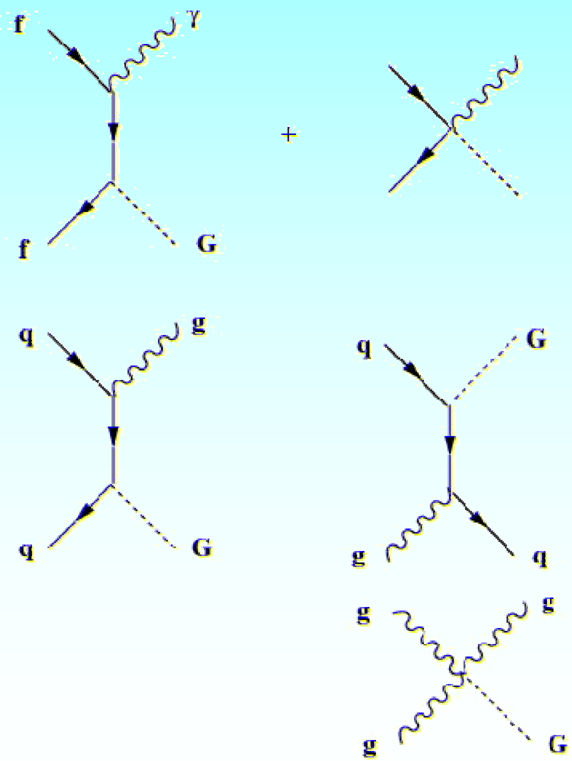
For ADD scenario with n extra dimensions $\lambda = R$ and $\alpha = \frac{8n}{3}$.



Detection 2: Graviton emmision into the bulk (Guidice, Rattazzi, Wells see e.g. C.Csaki hep-ph/0404096)

Gravity feels the extra dimensions - so we expect KK modes; each KK mode is emitted with 4-D gravitational strength.

Diagrams are



Hence $\Gamma \sim m^3/m_{Pl}^2$ and the graviton is lost so get e.g.

$$e^+e^- \to \gamma G_{KK}$$

or

$$q\bar{q} \to jet + G_{KK}$$

then $\sigma(E) \sim \frac{\alpha}{m_{Pl}^2} N(E)$. But $N(E) = (ER)^n$ so that get

$$\sigma \sim \frac{\alpha}{E^2} \left(\frac{E}{M_s} \right)^{n+2}.$$

When the energy reached the string scale this missing E_T becomes detectable.

e.g. Acosta et al (CDF) hep-ex/0205057. Run 1b ($87 \pm 4 pb^{-1}$ of $p\bar{p}$ collisions at $\sqrt{s} = 1.8TeV$). Selection on missing $E_T > 45GeV$. Backgrounds to $\gamma + missing\ E_T$ include $q\bar{q} \to \gamma Z \to \nu\bar{\nu}\gamma$ (3.2), cosmic ray muons (6.3) plus etc. Total is 11.0 ± 2.2, observed 11. At 95% C.L.

$$n = 4 \quad ; \quad M_s > 549GeV$$
$$n = 6 \quad ; \quad M_s > 581GeV$$
$$n = 8 \quad ; \quad M_s > 602GeV$$

Results similar to L3 and DELPHI;

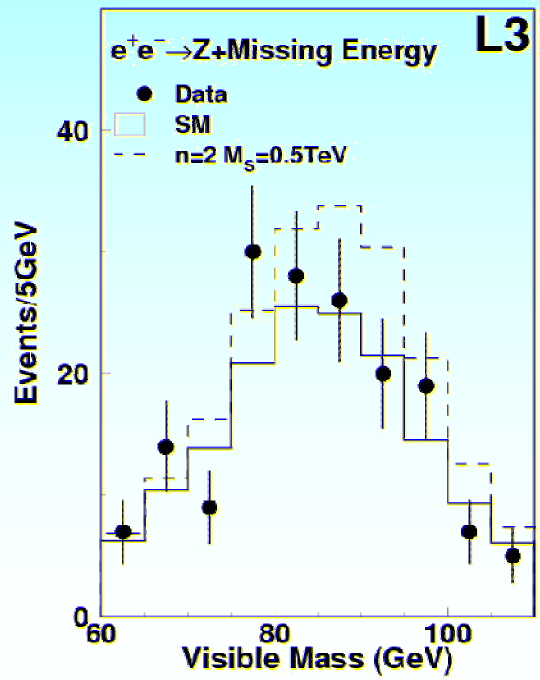


Figure 4: Visible mass for $e^+e^- \to ZG$ candidate events at 188.7 GeV together with SM expectations, dominated by W pair and single W production. The effect of real graviton production with two extra space dimensions and $M_S = 0.5$ TeV is also shown.

Detection 3: Virtual KK-graviton exchange (Guidice, Rattazzi, Wells, hep-ph/9811291)

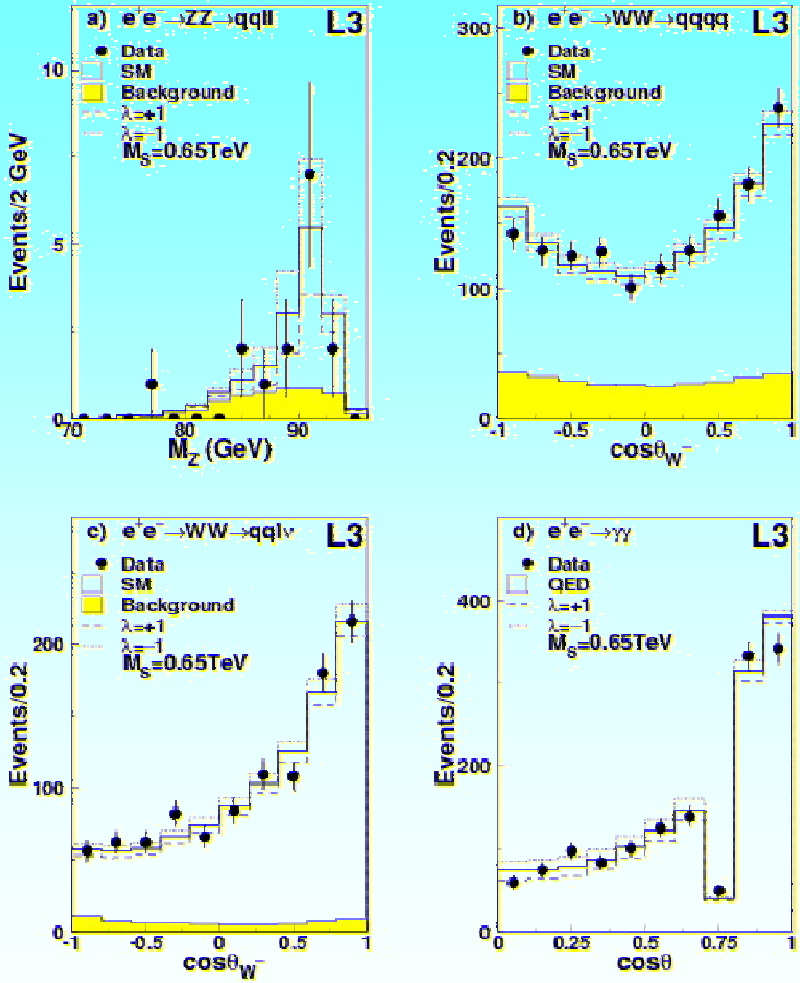
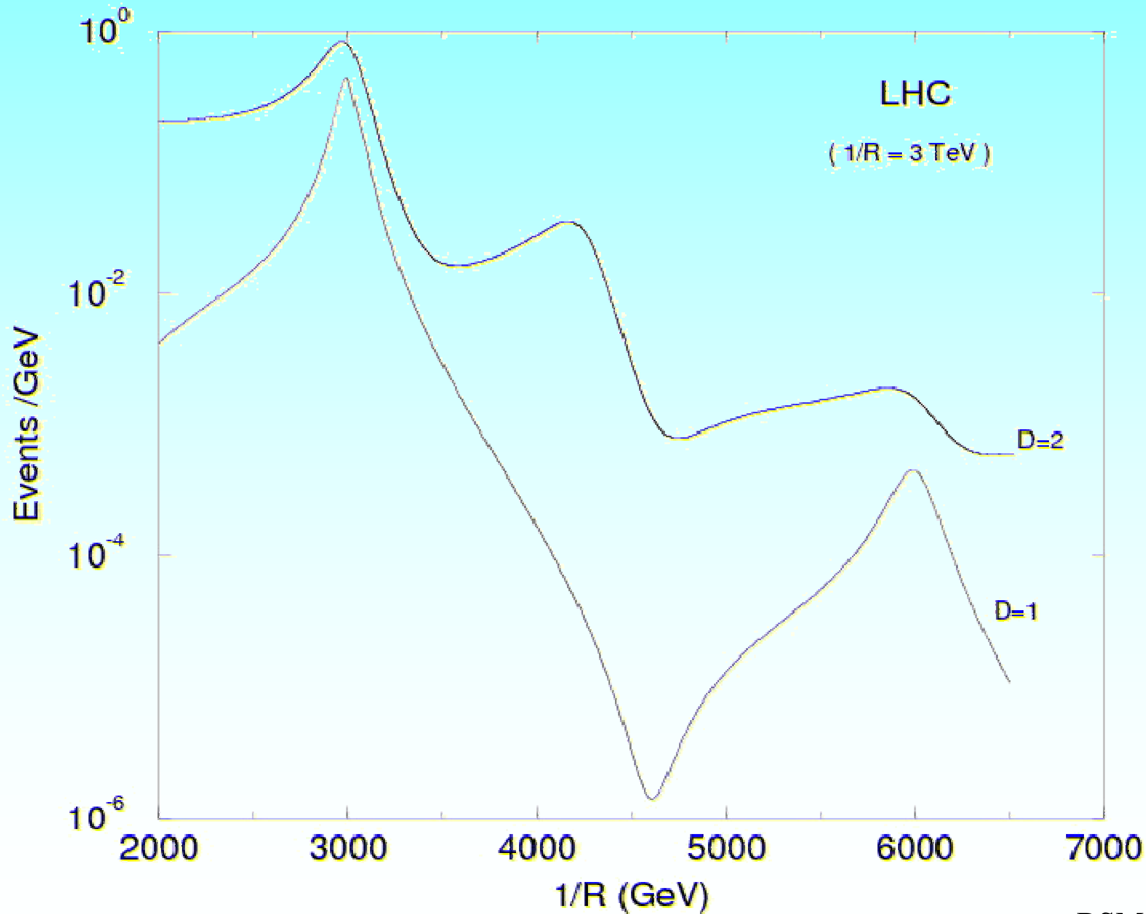


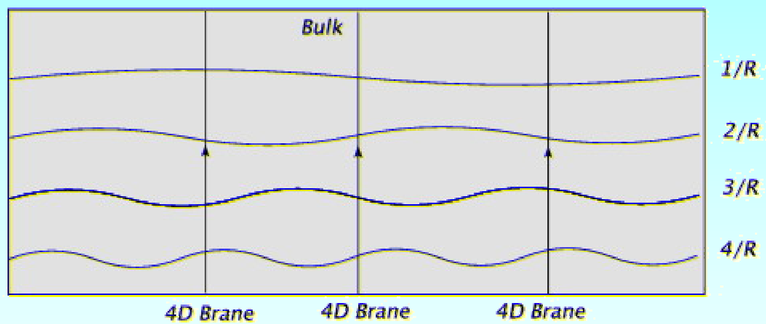
Figure 1: (a) Reconstructed Z mass for $e^+e^- \to ZZ \to q\bar{q}\ell^+\ell^-$ events. Distributions of the polar angle for: (b) hadronic $e^+e^- \to W^+W^-$ events, (c) semileptonic $e^+e^- \to W^+W^-$ events, (d) $e^+e^- \to \gamma\gamma$ events. Data at 188.7 GeV, SM signal and background expectations are presented together with LSG predictions for $M_S = 0.65$ TeV and $\lambda = \pm 1$.

Detection 4: Emission of string and KK-modes - we expect to find excitations of photons, Z's etc. e.g. if 1 or 2 radii are compact or if UED with a size a few TeV (0.1 Fermi) (Antoniadis, Benakli)



Detection 5: Flavour changing neutral currents

Return to extra dimension model and consider localized generations (split fermions). (Why? *a)* exponentially suppressed wavefunction overlaps can explain small Yukawa couplings (Mirabelli, Schmaltz. hep-ph/9912265), *b)* sometimes string models require this sort of set-up for consistency.)



$$L_5 = -\frac{1}{4} F^{MN} F_{MN} + i \overline{f}_i \gamma^\mu D_\mu f_i \delta(y - y_i)$$

- Go to the mass basis from the weak basis;

$$f_i = U_{ia} f_a$$

$$\mathcal{L}_{KE} \supset \sum_n A_\mu^{(n)} i \overline{f}_a \gamma^\mu U_{ai}^\dagger g_i^{(n)} U_{ib} f_b$$

At low energies, 4-fermion interactions are generated

$$\mathcal{L}_{effective} \supset c_{abcd} \left(\overline{f_a} \gamma^\mu f_b\right) \left(\overline{f_c} \gamma^\mu f_d\right)$$

where

$$c_{abcd} = -2g^2 \sum_{ij} U_{ai}^\dagger U_{ib} U_{cj}^\dagger U_{jd} \sum_m \frac{\cos\left(M_m (y_i - y_j)\right)}{M_m^2}$$

and $M_m = m/R$.

- Require $M_m > 100 TeV$ (Delgado Pomarol Quiros)

- But, note divergence problem with $n \geq 2$ extra dimensions;

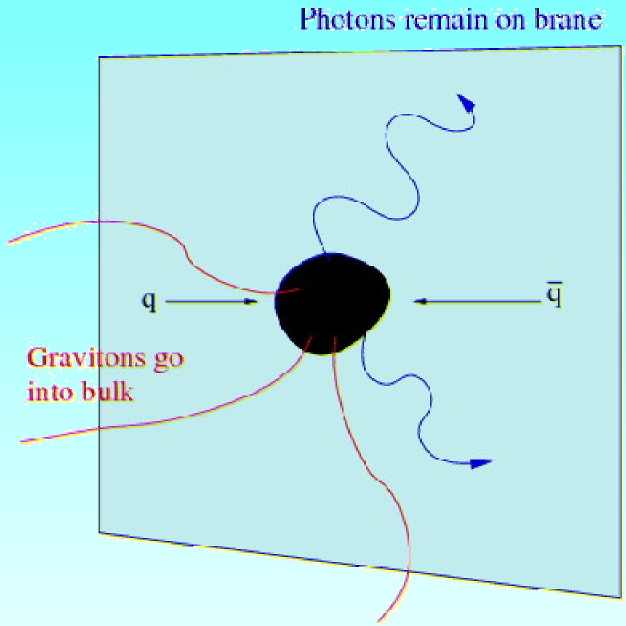
$$c_{abcd} \propto \sum_{\underline{m}} \frac{1}{M_{\underline{m}}^2} \sim R^2 \int d|m|\,|m|^{n-3}$$

physics sensitive to the UV completion

- The UV complete theory really does regulate it! In string theory

$$\mathcal{L} \sim g^2 \left[\bar{f}^i \gamma^\mu f^i \bar{f}^j \gamma_\mu f^j\right] \left(\frac{1}{s} + 2 \sum_m \frac{\cos\left(M_m (y_i - y_j)\right) \delta^{-M_m^2/M_s^2}}{s - M_m^2} \right)$$

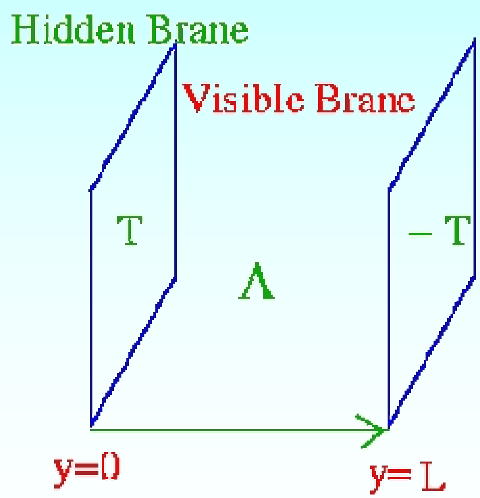
Detection 6: Detection of micro-Black-holes at e.g. LHC. (See Kanti hep-ph/0402168) (size $10^{-19}m$, lifetime $10^{-27}secs$)



- To first approximation, $\sigma \sim \pi R_s^2 \sim 1\, TeV^{-2} \sim 100\, pb$
- A significant proportion of the collision energy ends up inside the horizon
- Improvements; greybody factors - spectrum $\rightarrow n$.

RANDALL-SUNDRUM EXTRA DIMENSIONS

- In the previous approaches we considered a factorizable metric in $D = 4$, i.e. , $g_{\mu\nu}$ independent of extra dimensional coordinates y. That is not the most general situation.
- In fact branes themselves are *massive objects*, have tension T. Thus they may have an important gravitational effect on the surrounding geometry and create such a dependence.
- Randall and Sundrum considered in 1999 a particular 5-dimensional configuration with two 3-branes at the boundaries $y = 0$ and $y = \pi L$:



$$\Lambda = \frac{-T}{r}$$

- One finds that in such a configuration the metric

$$ds^2 = e^{-|y|/r}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

- solves 5-dimensional Einstein's equations. Here $\eta_{\mu\nu}$ is Minkowski metric and

$$r = \frac{24M_*^3}{T} \; ; \; \Lambda = \frac{-24M_*^3}{r^2}$$

- Now the metric felt at both branes is different:

$$g_{\mu\nu}{}^{vis} = e^{-L/r}\, g_{\mu\nu}{}^{hid}$$

- As a consequence all mass scales at the visible brane may be exponentially suppressed with respect to the natural scale $M_* \simeq 1/r \simeq M_{Planck}$. E.g.:

$$m_{Higgs}^2 = e^{-L/r} M_{Planck}^2$$

- Now, for $L/r \simeq 70$ one gets $m_{Higgs} \simeq$ 1 TeV providing a new solution to the hierarchy problem!

- Unlike the LED scenario here the Planck scale is of order of the fundamental scale M_*:

$$M^2_{Planck} = M^3_* \int_0^L dy e^{-y/r} = M^3_* r(1 - e^{-L/r})$$

- In the limit when $L \ll r$ one recovers the usual LED result for the Planck Mass:

$$M^3_* r(1 - e^{-L/r}) \longrightarrow M^3_* L$$

Solution to the hierarchy problem:

- Consider scalar Higgs fields ϕ trapped on the "weak" brane where $g^{ind}_{\mu\nu}|_{y=b} = e^{-2kb}\eta_{\mu\nu}$.

$$\begin{aligned} S^{Higgs} &= \int d^4x \sqrt{g^{ind}}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi^* - \lambda(|\phi|^2 - v^2)^2] \\ &= \int d^4x\, e^{-4kb}[e^{2kb}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi^* - \lambda(|\phi|^2 - v^2)^2] \\ & \int d^4x\,[\eta^{\mu\nu}\partial_\mu\hat{\phi}\partial_\nu\hat{\phi}^* - \lambda(|\hat{\phi}|^2 - (e^{-kb}v)^2]^2] \end{aligned}$$

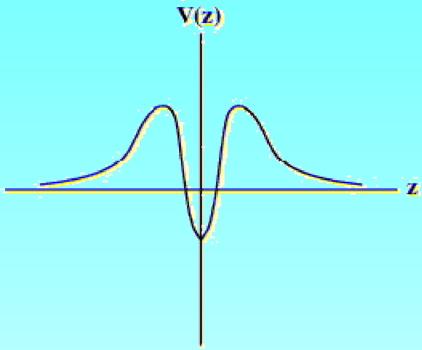
  where $\hat{\phi} = e^{-kb}\phi$.

- A "natural" 5-dimensional VEV of M_s is exponentially warped to $e^{-kb}M_s$ in 4-dimensional physics.

- Note that $m^2_{Pl} = M_s^3 \int_0^b e^{-2k|y|}dy = \frac{M_s^3}{k}(1 - e^{-2kb})$ is not changed hierarichally.)

Behaviour of graviton modes in RS1

- zero-mode (our 4D graviton!): sees a volcano potential and is localized near $y = 0$



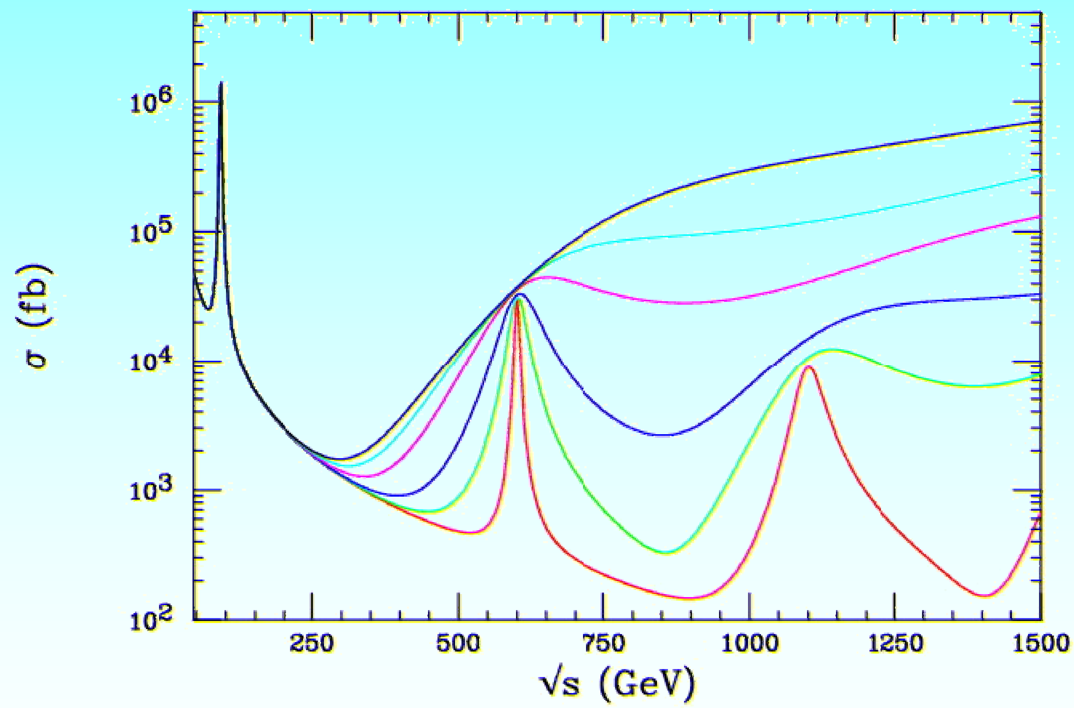
- Correction to Newton potential small for $r \gg M_s^{-1} \sim m_{Pl}^{-1}$;

$$V(r) = G_N \frac{m_1 m_2}{r}(1 + \frac{C}{(kr)^2})$$

- Because of warping **KK** modes have typical mass splittings of TeV^{-1}!
- But their wave functions are exponentially localized on weak brane - hence coupling suppressed by only TeV^{-1} not m_{Pl}^{-1}!

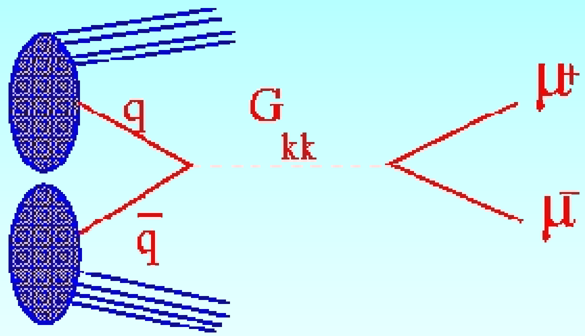
Detection

e.g. contribution to $e^+e^- \to \mu^+\mu^-$ (Davoudiasl, Hewett, Rizzo, hep-ph/9909255). First mode at 600GeV, $k/m_{Pl} = 1, 0.7, 0.5, 0.3, 0.2, 0.1$ top (black) to bottom (red)



RANDALL-SUNDRUM PHENOMENOLOGY

- There are Kaluza-Klein copies of the graviton but unlike the LED scenario they have TeV masses.
- Due to that fact constraints from deviations from Newton's law and astrophysics essentially disappear
- These KK TeV gravitons may be produced a la Drell-Yan at Tevatron and LHC:



- There is also a scalar ϕ, the *radion* whose vev determines the brane separation, $\phi = L$. It is expected to have a mass in the range $0.1 - 1$ TeV.
- It may be produced by gluon fusion and decays into a dijet or ZZ.

- A number of variations of the basic RS scenario have been proposed.
- If an extra scalar is postulated to live in the bulk the vev for the *radion* (and hence the size of extra dimensions) may be dynamically determined, [a].
- RS scenarios with supersymmetry [b] and/or grand unification [c] have been explored.
- It has also been argued that perhaps the logarithmic running of gauge couplings still holds even though there is no longer a big dessert, due to the warped metric [d].
- More interestingly Randall and Sundrum also proposed a different scenario with warping and only one 3-brane in which gravity is dynamically localized on a 3-brane and compactification is not required!
- A lot of possibilities and an embedding in string theory are actively pursued at present.

---

[a]Goldberger and Wise 1999

[b]Bagger et al.Luty,Sundrum; Bagger et al (2001)

[c]Pomarol, (2000)

[d]Randall, Schwartz (2001)

SOME COMMENTS ON EXTRA DIMENSIONS

- Perhaps the most exciting feature of the extra dimensions scenario is the possibility that they could be around the corner, even accesible to accelerators.
- The gauge hierarchy problem is translated into a dynamical question, i.e., finding why some of the extra dimensions are very large compared to the fundamental scale.
- Some nice properties of the SUSY/GUT scenario are lost: gauge coupling unification, simple explanation for proton stability...
- Building a completely successfull cosmology (inflation, baryogenesis) may also prove hard
- On the other hand GUT's with some extra dimensions could be usefull to overcome the doublet-triplet problem
- The Randall-Sundrum idea provides a complete new way to understand the hierarchy between M_W and M_{Planck} scales
- An important remark: theories of extra dimensions are non-renormalizable, there should be some more fundamental theory which renders the theories consistent
- The most natural candidate for such a more fundamental theory is STRING THEORY

Extra dimension perspectives

Models with extra dimensions can

- unify all interactions
- solve the hierarchy problem
- link String Theory to Standard Model
- make Quantum Gravity and String Theory accessible at LHC
  like Prometeus made the divine fire accessible for people

Perhaps it is only a dream...

But I wish you and me this dream to come true!